

A Closed Form Solution for Multiple-Input Spike Based Adaptive Filters

Il Park, António R. C. Paiva, José C. Príncipe and John G. Harris

Abstract—Neurons are point process systems, in the sense that the inputs and output which are spike trains can be treated as point processes. System identification of a point process system has been studied mostly with single input. However, multiple input is required in many applications such as liquid state machines or neural prosthetics. We propose a simple multiple-input spike based adaptive filter which is based on an integrate-and-fire neuron model. The optimal closed solution is derived, and the performance is analyzed with respect to noise in various parameters and measurement.

I. INTRODUCTION

Biological neuronal networks utilize both continuous amplitude signals and point process signals for computation. The signals are almost always greatly undersampled due to the enormous number of signal sources and limitations of the measurement technology. In many cases, only spike trains are available because spike trains have high signal to noise ratios. Thus, neuronal systems can be best modeled as point process systems. System identification of such systems has various applications such as neural prosthetics [1], system identification of a single neuron [2], modeling neuronal networks [3], designing a natural readout for liquid state machines [4], and calibration of neuromorphic VLSI [5]. In order to obtain the analytical solution for the system identification problem that would provide insight into the problem, we use a simplified model which deviates from standard biological models. In addition, it will provide a baseline for future studies.

There has been extensive work done in identifying point process systems. Traditional adaptive signal processing techniques, such as Volterra series [6] or least mean squares filter methods [7], can be applied to point processes by discretizing the time to get the input signals and thresholding the output signal to get spike train outputs. An alternative approach is to use a frequency domain estimation of the intensity functions assuming Poisson statistics [8], [3], [9]. For continuous input to point process output, neuron models are usually incorporated [10], [2]. Although some of the previous work can be extended to multiple inputs of point processes, there is not much work specifically addressing this goal. Current multielectrode recording technology provides the opportunity to estimate the spatial organization of the inputs to a neuron or a neuronal network (cf. [11]) rather than synaptic transfer function or neuronal response to a current injection.

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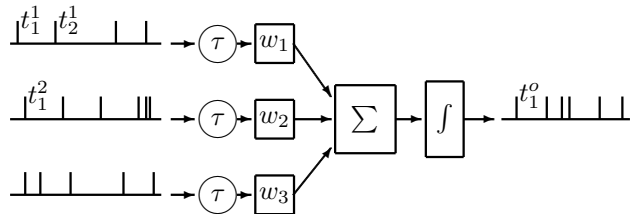


Fig. 1. Schematic diagram for the architecture of the filter for three input spike trains.

The goal of this paper is to use multiple spike trains as inputs to a neuron model that generates precise spike timing. In order to derive an analytical optimal solution, we use an integrate-and-fire (IF) neuron with a static linear synapse. As a result of this model and our cost function, a Wiener-like solution to estimate the synaptic weights is obtained.

II. MODEL

The architecture of the model consists of two parts: 1) conversion of input spike train to continuous synaptic current, and 2) integration of current and thresholding to form an output spike train (Fig. 1). For an ideal IF neuron, the integral of the input current between two consecutive firings, denoted as t_i^o and t_{i+1}^o , equals the threshold,

$$\int_{t_i^o}^{t_{i+1}^o} I(t)dt = \theta \quad (1)$$

where $I(t) \geq 0$ is the input current, θ is the threshold voltage. When the IF neuron fires, the membrane potential and all synaptic input currents are reset (single compartment model). This system wide reset erases the effect of history, thus the next output action potential is only dependent on the input spikes received after the previously generated action potential.

In the simplest case, all synapses have identical time-invariant dynamics and can be represented as a causal linear filter h and a scaling factor (synaptic weight) of $w_j \geq 0$. Then, the individual post-synaptic current $q_j(t)$ and total input current $I(t)$ will be represented as:

$$q_j(t) = \sum_{t_i^o \leq t_k^j} h(t - t_k^j), \quad (2)$$

$$I(t) = \sum_j w_j q_j(t), \quad (3)$$

where t_i^o is the last output spike of the neuron, t_k^j is the k -th spike time of j -th input spike train, and the summation in

q_j is over valid k , so that only the input spikes after the last output spikes are summed. Now the membrane potential is

$$v(t) = \sum_j w_j \int_{t_i^o}^t q_j(\tau) d\tau. \quad (4)$$

Let us simplify the integral,

$$\int_{t_i^o}^t q_j(\tau) d\tau = \int_{-\infty}^t \sum_{t_i^o \leq t_k^j \leq t} h(\tau - t_k^j) d\tau \quad (5)$$

$$= \sum_{t_i^o \leq t_k^j \leq t} H(t - t_k^j) \quad (6)$$

where we define

$$H(t) = \int_{-\infty}^t h(\tau) d\tau. \quad (7)$$

Now the membrane potential is simplified to,

$$v(t) = \sum_j w_j \sum_{t_i^o \leq t_k^j \leq t} H(t - t_k^j). \quad (8)$$

We used a simple exponentially decaying synapse,

$$h(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} u(t) \quad (9)$$

$$H(t) = (1 - e^{-\frac{t}{\tau}}) u(t) \quad (10)$$

where $u(t)$ is the Heaviside function. However, any type of synapse can be used without much effort. The time constant for each synapse can be different, but for simplicity of the presentation of the method they are fixed to be the same. For simulation, Newton's method is used to find the precise firing times of the IF neuron.

III. COST FUNCTION

For a given desired output spike train, the membrane potential has the following constraints:

$$v(t) < \theta, \quad \text{for } t_i^o < t < t_{i+1}^o \quad (11)$$

$$v(t) = \theta, \quad \text{for } t = t_i^o \quad (12)$$

Since in IF neurons, $v(t)$ is a monotonic increasing function between consecutive action potentials, the second constraint is sufficient to ensure the first constraint.

Let us solve this by minimizing the mean square error (MSE) defined for the period between each pair of action potentials. The MSE cost function can be estimated with M desired output spikes t_i^o ,

$$J = \frac{1}{2} \frac{1}{M} \sum_{i=0}^{M-1} (v(t_{i+1}^o) - \theta)^2 \quad (13)$$

$$= \frac{1}{2} \frac{1}{M} \sum_{i=0}^{M-1} \left(\sum_j w_j \sum_{t_i^o \leq t_k^j \leq t_{i+1}^o} H(t_{i+1}^o - t_k^j) - \theta \right)^2, \quad (14)$$

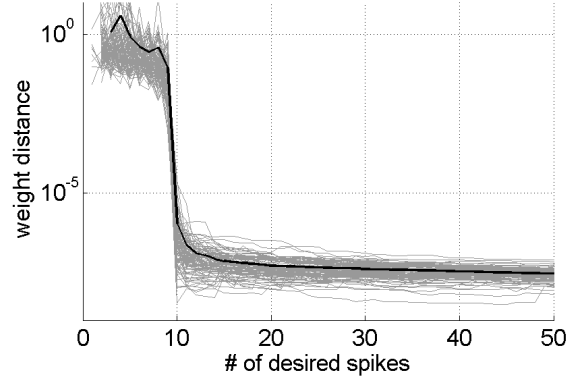


Fig. 2. Euclidean distance between true weight vector and estimated weight vector in log scale over the number of desired output spikes used for the estimation. For less than 10 desired spikes, the estimation is highly unstable, but after that the estimated weight converges rapidly. Monte Carlo simulation is performed for 200 runs for each number of desired spikes. The number of input spike trains was 10, the mean input firing rate was 12 Hz, the threshold was 0.1, synaptic time constant was 5 ms, and the true weights were uniformly distributed between 0 and 0.01.

where $t_0^o = 0$. Since H , t_k^j , and t_i^o are all known, it can simply be written as,

$$J = \frac{1}{2} \frac{1}{M} \sum_{i=0}^{M-1} \left(\sum_j w_j \alpha_{ij} - \theta \right)^2, \quad (15)$$

where $\alpha_{ij} = \sum_{t_i^o \leq t_k^j \leq t} H(t_i^o - t_k^j)$ (the summation is over the valid range of k). $w_j \alpha_{ij}$ is the contribution of j -th neuron to the membrane potential after i -th output spike at time of the $i + 1$ -th output spike.

IV. SOLUTION

By letting the partial derivative of the cost function with respect to weights w_k to be 0, we can find the minimum of the quadratic performance surface.

$$\frac{\partial J}{\partial w_k} = \frac{1}{M} \sum_{i=0}^{M-1} \left(\sum_j w_j \alpha_{ij} - \theta \right) \alpha_{ik}. \quad (16)$$

Thus, the optimal solution satisfies the following equation,

$$\sum_i \left(\sum_j w_j \alpha_{ij} - \theta \right) \alpha_{ik} = 0. \quad (17)$$

Define a matrix $A_{jk} = \sum_i \alpha_{ij} \alpha_{ik}$, and a vector $B_k = \sum_i \theta \alpha_{ik}$ then,

$$\mathbf{A} \vec{w} = \vec{B}, \quad (18)$$

which can be solved by using the Moore-Penrose pseudo inverse \mathbf{A}^\dagger ,

$$\vec{w} = \mathbf{A}^\dagger \vec{B}. \quad (19)$$

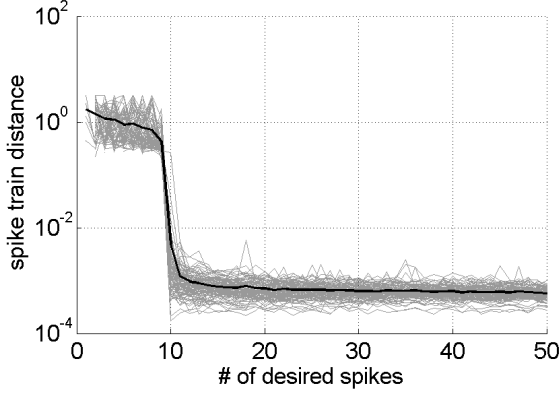


Fig. 3. Distance between output spike train generated with true weights and estimated weights measured with a slightly modified distance measure of [12] with a time constant of 50 ms (normalization with $\frac{1}{M}$ in equation (2.2)). Same Monte Carlo runs as Fig. 2.

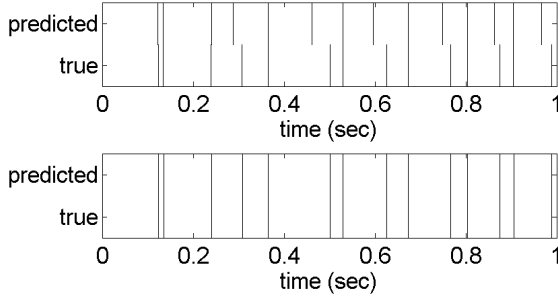


Fig. 4. Comparison of output spike trains from true weight and estimated weight. The weights were estimated with 66 desired spikes, and the precision of spike timing was 5 ms (top) and 1 ms (bottom). This corresponds to a sampling rate of 200 Hz and 1 kHz.

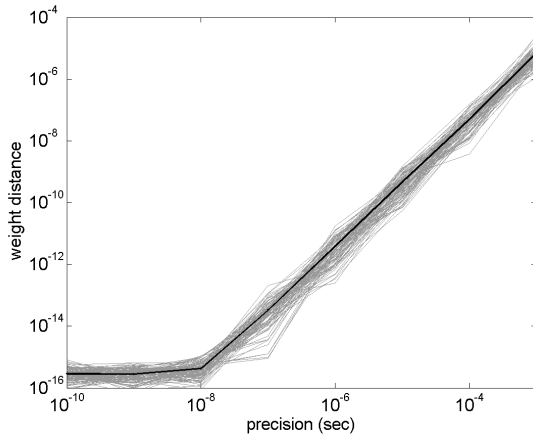


Fig. 5. Effect of spike time precision to error in weight estimation. Error in weight linearly decreases as precision increases up to 10 ns. The simulated spike times using Newton's method with a given precision upper bound with Poisson input and 20 desired output spikes. The average is plotted as black, and 100 Monte Carlo runs are plotted as gray.

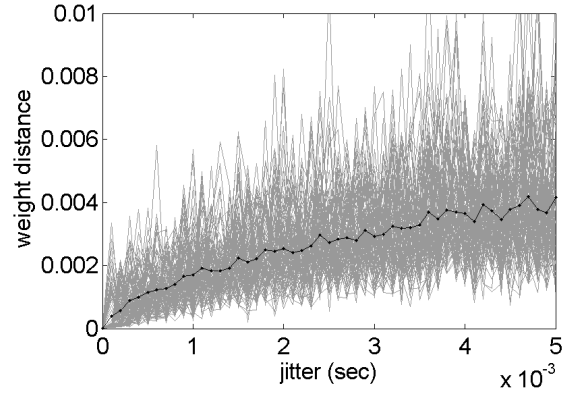


Fig. 6. Effect of noisy jitter in input spike timings. The error in weight versus standard deviation of Gaussian noise is plotted. Precision was 10^{-5} , and 20 desired output spikes was used for the estimation. Again, average is plotted as black, and 100 Monte Carlo runs are plotted as gray.

V. ANALYSIS

Practically, no system parameter or measurement can be infinitely precise. In this section, we analyze the performance of the estimator for various sources of noise. For all computation, mutually independent sets of homogeneous Poisson input spike trains were used as input. The number of input spike trains is 10, the average firing rate is 12 Hz, and the synaptic time constant τ is 5 ms, if not stated otherwise.

A. Number of desired spikes

The number of unknowns in the system is determined by the number of input spike trains N . Since, the number of rows in A is determined by the number of desired output spike M that is used for training, (18) is underdetermined if $M < N$. Even when it is overdetermined, depending on the space spanned by the input spike trains, the estimate is not perfect, and the error of estimate will decrease as more desired spikes are used. In Fig. 2, the error is measured as the error in weight vector estimation, and the drop of the mean and variance of the error after $M = 10$, the number of unknowns, is clearly observed.

We can also measure the error with the distance between the output spike trains of a system with the true weights and the estimated weights for the same novel set of input spike trains. In Fig. 3, a similar trend can be observed as the weight error. However, the testing involves using Poisson input spike trains, which increases the variance. Thus, we will use weight distance to measure the estimator performance.

B. Precision of spike timings

The precision of spike times is fundamentally restricted by sampling rate of the recording device. In Fig. 4, two examples of prediction for the output spike train are shown. Fig. 5 shows the improvement in weight estimation for improvement in spike time precision. Note that the error linearly decreases as precision increases.

In the level of (biological or silicon) neuron, there is noise in the membrane potential and action potential generation

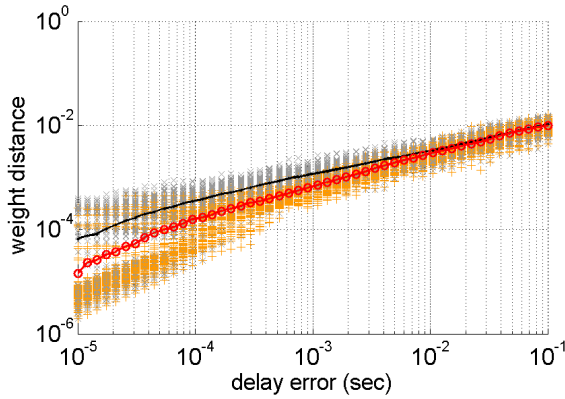


Fig. 7. Effect of constant delay in input spike train in log-log scale. The positive delay is indicated by the black solid line, and the negative delay is indicated by the red circled line (the gray and orange corresponds to the Monte Carlo instances respectively).

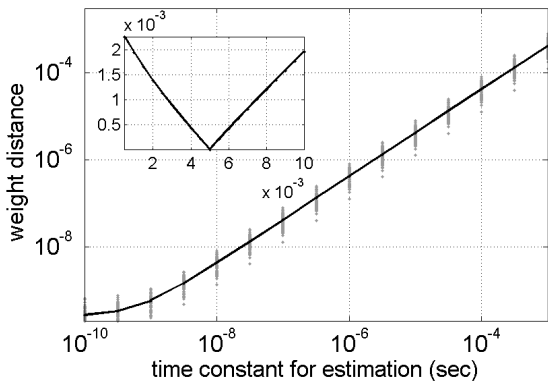


Fig. 8. Effect of time constant τ mismatch. The relative amount of mismatch in τ versus weight distance is plotted. (inset) same plot for τ in [0.5, 10] ms. The actual value used to generate the training data is 5 ms.

mechanism (threshold). This can be approximated by putting jitter to individual spike timings of the input. Fig. 6 shows the error in weight over standard deviation of the noise. Jitter in the output spike train also has a similar trend (data not shown).

C. Delay and time constants

In a biological system, there are three major sources of delays for a neuron: 1) the conduction delay through the axon and synaptic delay, 2) the shape of post synaptic potential which effectively contributes to the firing probability distribution in a causal manner, 3) the propagation delay through the dendrite which can be modeled as cable, and distorts the shape of the post synaptic potential. The model in this paper is restricted to have only delays of type 2).

By shifting all input spike trains with a constant time, the error due to the shift is nearly linear for large delays as shown in Fig. 7. It is notable that negative delays (input precedes output) have less error. For positive delays, the first spike that is going to be lost is the last spike, which contributes

The fact that for positive delays it is easy to lose the last spike, that generated the desired output, but for negative de-

lays the initial spikes which saturate and does not contribute to the actual firing could be the reason.

For the time constant τ mismatch, Fig. 8 suggests the correct value of τ is strongly preferred. Since the slope is almost linear, it is possible to estimate τ in a straight forward manner.

VI. CONCLUSION

The optimal closed form solution for identifying weights in a simple point process system is presented. The method provides means to estimate synaptic weights from a short segment of a spike train and it is computationally efficient since it only depends on the number of spikes, no matter how long the spike train is. This is possible from the fact that in the model the membrane potential is a non-decreasing function during the time between a pair of output spikes. However, due to the deterministic nature of the model, it is highly sensitive to many parameters, as demonstrated in the figures. Thus, it is necessary to estimate the other parameters accurately, and also have high temporal resolution spike trains. Generalization of the method to estimate the synaptic time constant and conduction delay is necessary to apply to recorded data.

ACKNOWLEDGMENTS

I. P. thanks Karl Dockendorf and Jie Xu for insightful discussion. This work was partially supported by NSF grant ECS-0422718. A. R. C. Paiva was supported by Fundação para a Ciência e a Tecnologia under grant SFRH/BD/18217/2004.

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