Reproducing kernel Hilbert Spaces for Spike Train Analysis

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Outline

- Motivation and goals
- Generalized Cross-Correlation
- Instantaneous Cross-Correlation
- Results

Motivation

- Need a mathematical framework for the analysis of multiple spike trains.
- The fundamental concept for data analysis is the definition of an inner product.

$$\Downarrow$$

- The mathematical structure needed for analysis is available in the form of Reproducing Kernel Hilbert Spaces (RKHS).

Motivation

- Cross-correlation is widely used for spike train analysis, but there are four main limitations:
  1. Traditionally, it is applied to binned spike trains
  2. It assumes stationarity (and ergodicity)
  3. To deal with non-stationarity the analysis is done in moving windows, with a tradeoff between temporal resolution and estimation accuracy
  4. Analysis only for pairs of spike trains.

Goals

- Define a Reproducing Kernel Hilbert Space (RKHS) based on ideas for cross-correlation, to remove or alleviate these limitations
  - Binning-free and data efficient estimation
  - High-temporal resolution exploring the spatial dimension of multi-electrode recordings and capable of cope with nonstationarity.

What is Cross-Correlation?

- Cross-correlation is an inner product

$$C_{AB}^{bin}[l] = \frac{1}{M}\sum_{n=1}^{M} N_A[n] N_B[n+l]$$

  - The mapping into the RKHS is binning, which maps time to amplitude randomness, but loses information since time is quantized
- What is the role of binning?
- Can an inner product be defined without these limitations?

Generalized Cross-Correlation

- Binning is an intensity estimator!
- Hence, using the intensity functions of the underlying point processes we can write a generalized cross-correlation (GCC),

$$C_{AB}(\theta) = E\left\{\lambda_A(t)\lambda_B(t+\theta)\right\} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\lambda_A(t)\lambda_B(t+\theta)dt$$

  - This is a functional inner product using the full resolution of the recordings
  - But, it still assumes stationarity (and ergodicity)

Generalized Cross-Correlation

Estimation from data

- To estimate this inner product, we start by estimating the intensity function using kernel smoothing

$$\hat{\lambda}_A(t) = \sum_{m=1}^{N_A} h(t - t_m^A)$$

- Substituting is yields the estimator

$$\hat{C}_{AB}(\theta) = \frac{1}{T} \sum_{m=1}^{N_A} \sum_{n=1}^{N_B} \kappa_\tau \left(t_m^A - t_n^B + \theta \right)$$

Generalized Cross-Correlation

Advantages

- Cross-correlation is a special case of the GCC, in which spike times are quantized and a rectangular function is used for κ_τ
- Operates directly on the spike times, utilizing the full resolution of the recordings
- Takes advantage of the sparse nature of spike trains
- Smoothing by κ_τ allows for multiscale analysis, between synchrony or firing rate

Instantaneous Cross-Correlation

- But what about high temporal resolution?
  - GCC still assumes (piecewise) stationary analysis
- Solution: drop the expectation over time!

- Then, the instantaneous cross-correlation (ICC) is defined as

$$\tilde{c}_{AB}(t,\theta) = \hat{\lambda}_A(t)\hat{\lambda}_B(t+\theta).$$

Instantaneous Cross-Correlation

Online estimation

- Estimation is very simple, even online…
- For online estimation, consider

$$h(t) = (1/\tau)\exp\left[-t/\tau\right] u(t),$$

  as the smoothing function.
- The estimated intensity function is

$$\hat{\lambda}_A(t) = \frac{1}{\tau} \sum_{t_m^A \leq t} \exp\left(-\frac{t - t_m^A}{\tau}\right) u(t - t_m^A).$$

Instantaneous Cross-Correlation

Approximating the GCC

- $\tilde{c}_{AB}(t,\theta)$ is a stochastic approximation of the GCC,

$$\frac{1}{T}\int_0^\infty \tilde{c}_{AB}(t,\theta)dt = \frac{1}{T}\sum_{m=1}^{N_A}\sum_{n=1}^{N_B}\frac{1}{2\tau}\exp\left(-\frac{|t_m^A - t_n^B + \theta|}{\tau}\right)$$
$$= \frac{1}{T}\sum_{m=1}^{N_A}\sum_{n=1}^{N_B}\kappa_\tau\left(t_m^A - t_n^B + \theta\right)$$
$$= \hat{C}_{AB}(\theta),$$

Instantaneous Cross-Correlation

As a neural ensemble measure

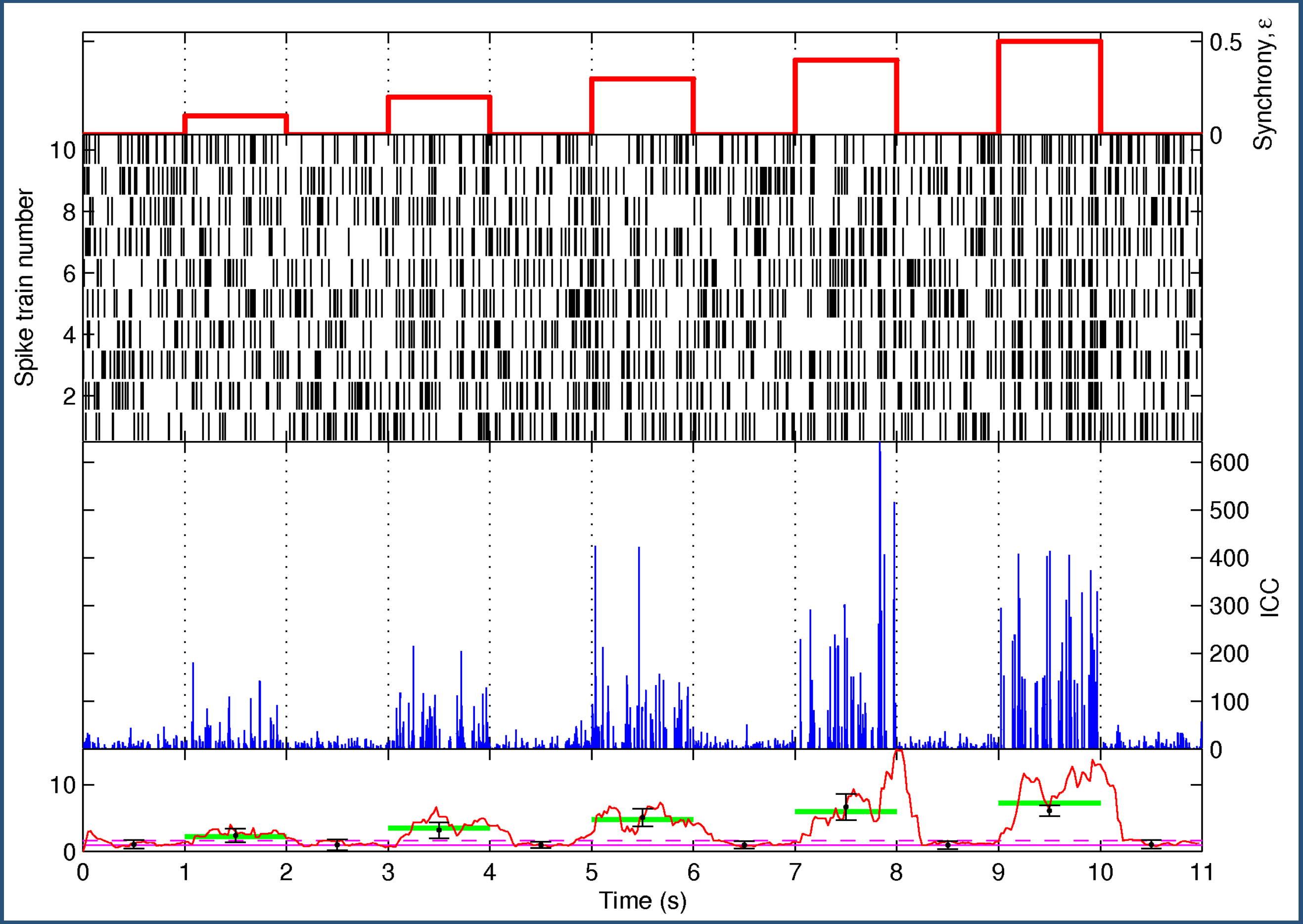
- To achieve high temporal resolution, the expectation over time was dropped, which the variance of the measurements…
- Solution: compute the expectation over the ensemble!

$$\bar{c}(t,\theta) = E\left\{\tilde{c}_{AB}(t,\theta)\right\}$$

- Giving rise to a spatio-temporal measure of correlations across the ensemble.

Results

ICC as a synchronization measure



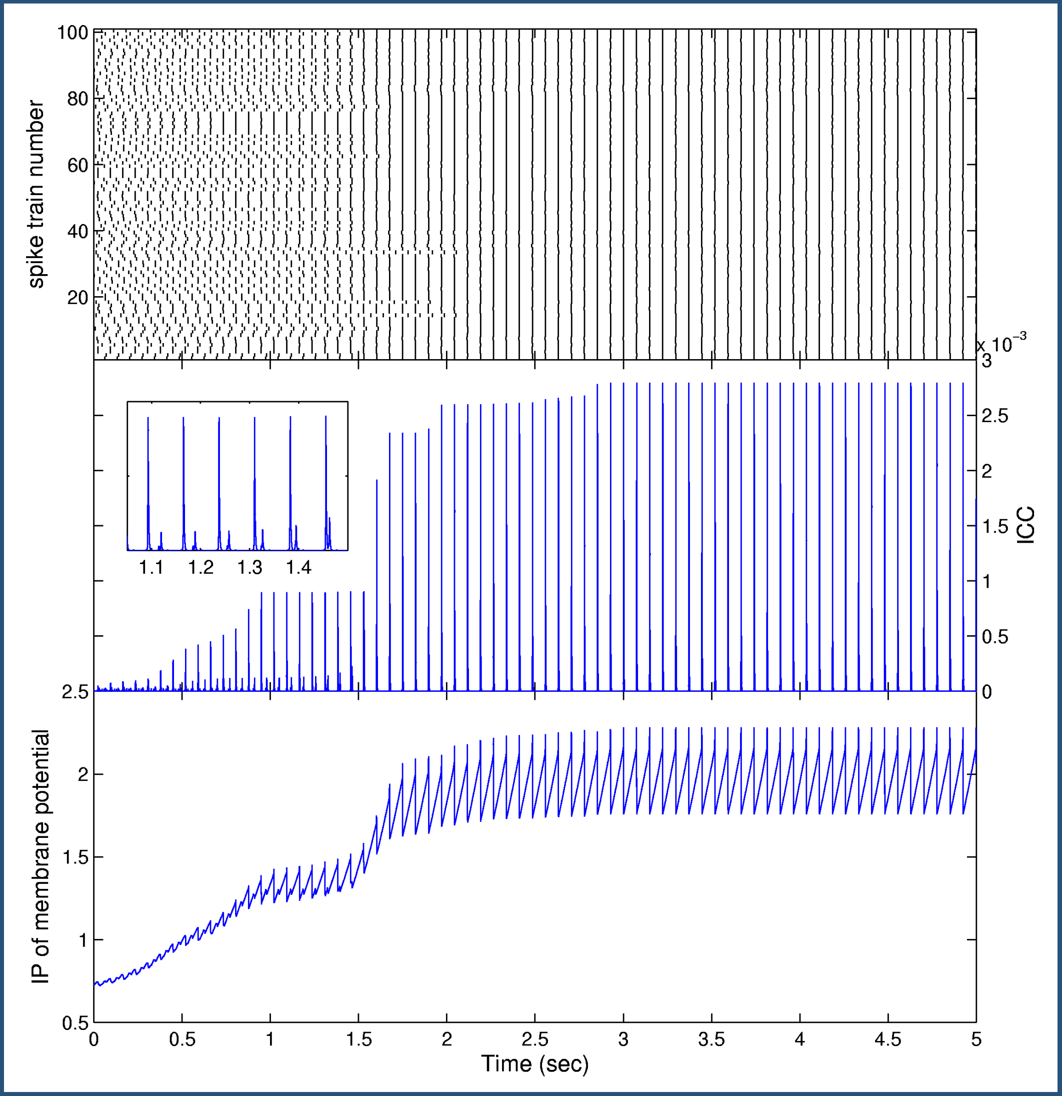
Results

Synchronization of pulse-coupled oscillators

- Designed a spiking neural network of leaky integrate-and-fire neurons
- Network is known to synchronize over time (Mirollo & Strogatz, 1990)
- Utilized ICC to verify the evolution of synchrony
- Compared to an information-theoretic measure of internal coherence (membrane potentials)

Results

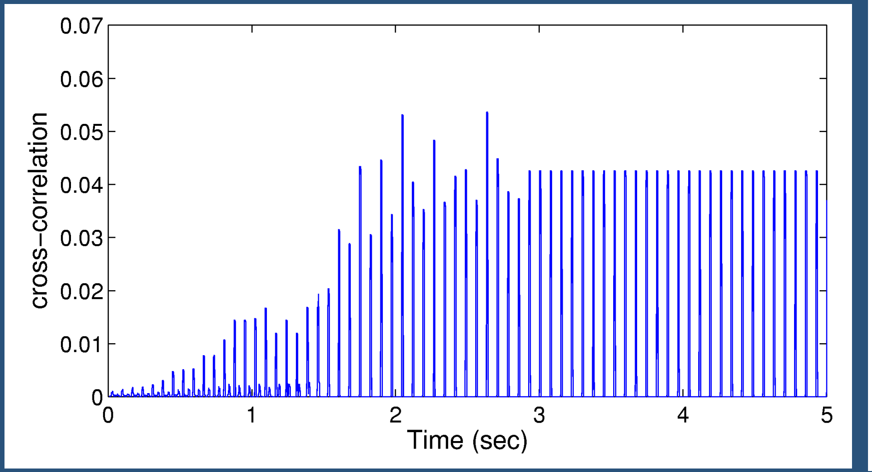
Synchronization of pulse-coupled oscillators

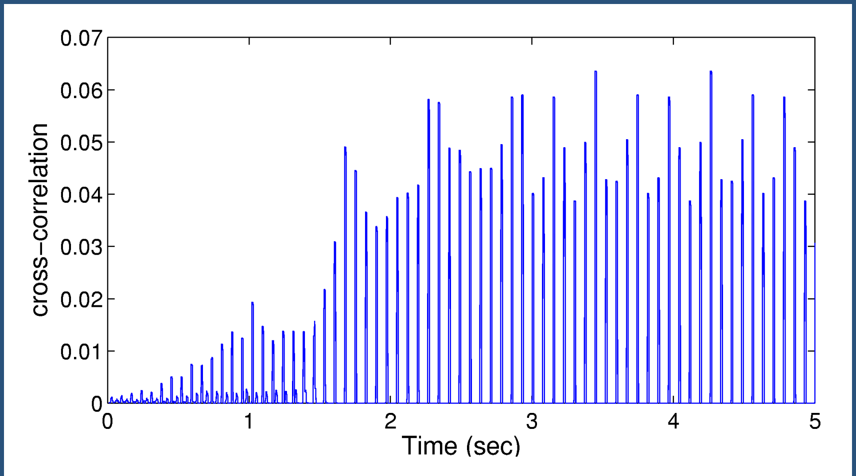


Results

Synchronization of pulse-coupled oscillators

- Sensitivity of cross-correlation to bin size





Conclusion

- Utilized RKHS threory ideas to define GCC as an inner product of spike trains.
  - Computationally simple and accurate estimator
  - Natural extends towards multiscale analysis
- Suggested the ICC as a high temporal resolution measure of ensemble couplings.
  - Trades time averaging for “spatial” averaging
  - But, requires knowledge of groups of neurons over which to average

Future work

- Work on clustering and PCA of spike trains to find the ensembles of neurons over which to average the ICC.
- Most important of all, the RKHS allows for the development of more principled algorithms for computation with spike trains.