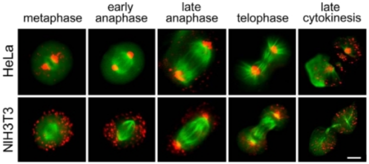
Utilizing Topological Data Analysis to Detect Periodicity

Elizabeth Munch

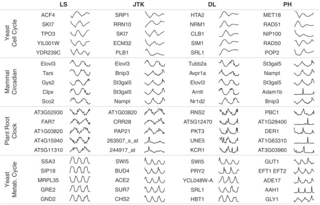
University at Albany - SUNY :: Department of Mathematics & Statistics

Oct 2, 2016

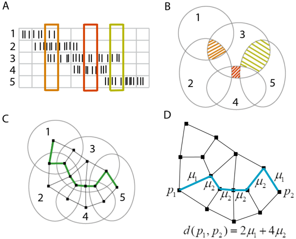
Time series in biology



Mitosis
Kredel et al. PLoS One 2009



Yeast gene expression
Deckard et al., Bioinformatics 2013



Neuron Spike Trains
Curto et al. PLoS One 2008

ECG
Goldberg et al. 2000

Our definition of time series

Definition

A time series is a function

$$f : \mathbb{R}_{\geq 0} \longrightarrow D$$

for some topological space D.

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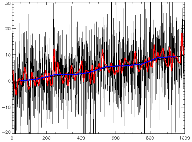
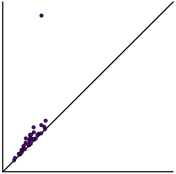
Choice for D

- $\mathbb{R}$ - Classical time series analysis
- $\mathbb{R}^{m \times n}$ - $\mathbb{R}$-valued $m \times n$ matrices (movies)
- Pers - Persistence diagram valued time series (vineyards)

Commonly used tools





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\mathbb{R}\text{-valued TS} \arrow[r, "\text{Takens Embedding}"] & \text{Pers} \\
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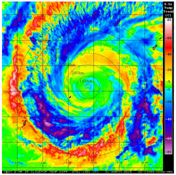

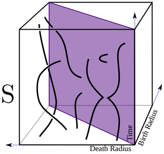

Common questions

- Classification/Clustering
  - Is this signal Type A or Type B?

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Idea:

Persistent homology and other TDA tools can be used to improve time series anaysis.

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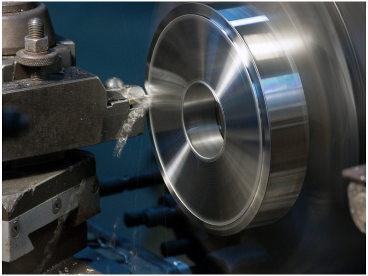
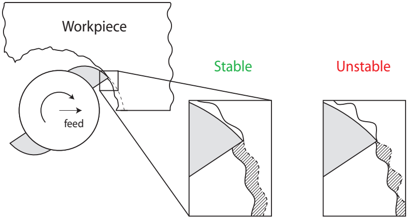
Persistent homology and other TDA tools can be used to improve time series anaysis.

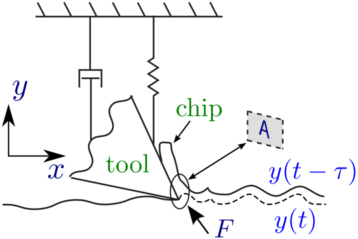
This talk:

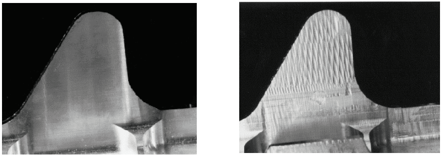
- Mechanical engineering
  - Firas Khasawneh
  - Jose Perea
- Atmospheric science
  - Bill Dong
  - Kristen Corbosiero
  - Jason Dunion
  - Ryan Torn

Machining Dynamics





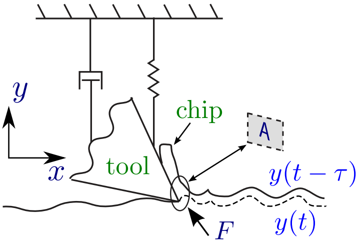




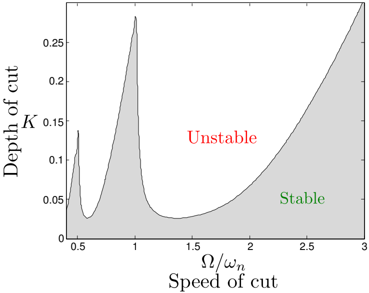
Images courtesy Firas Khasawneh, SUNYIT; and Boeing.

Deterministic model:

$$\ddot{y} + 2\zeta\dot{y} + y$$
$$= K\rho^{\alpha-1}(1 + y(t-\tau) - y(t))^{\alpha}$$

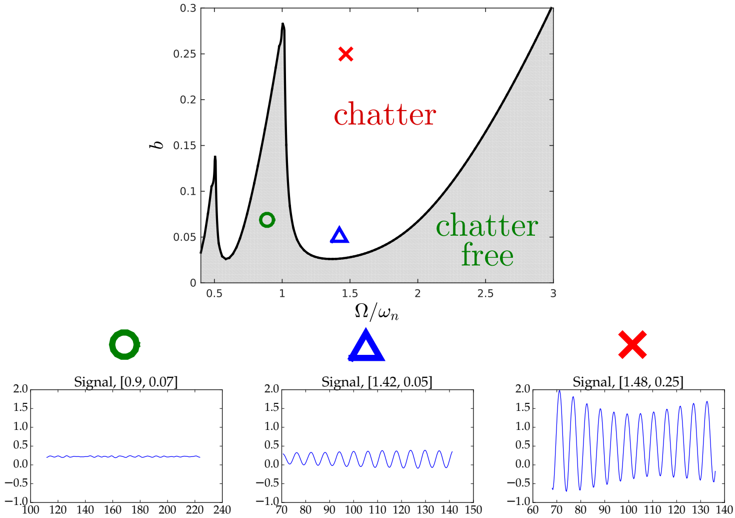


- Left side: standard linear oscillator
- Right side: input based on cutting forces



Khasawneh, F.A. & Mann, B. P. A spectral element approach for the stability of delay systems, International Journal for Numerical Methods in Engineering, 2011, 87, 566-592

Chatter



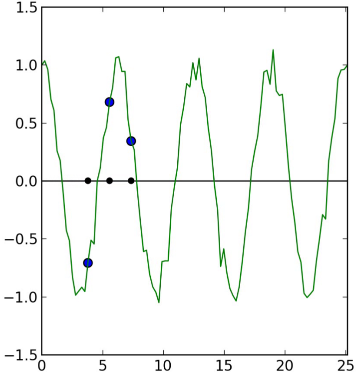
chatter
chatter free
b
Ω/ω_n
Signal, [0.9, 0.07]
Signal, [1.42, 0.05]
Signal, [1.48, 0.25]

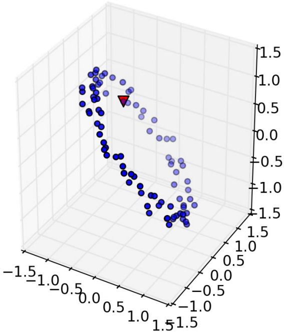
Takens embedding

Definition

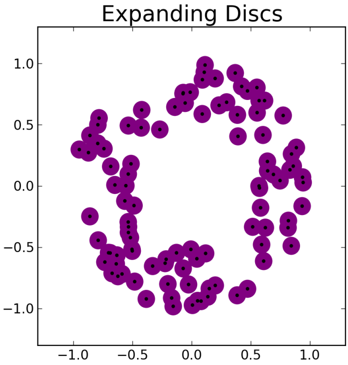
Given a time series $X(t)$, the Takens embedding is

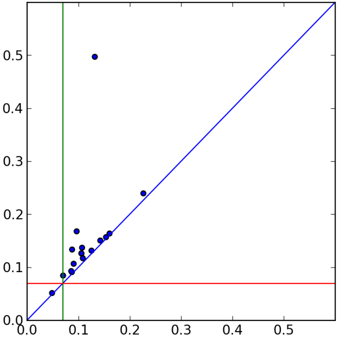
$$\psi_\eta^m : t \longmapsto (X(t), X(t+\eta), \cdots, X(t+(m-1)\eta)).$$



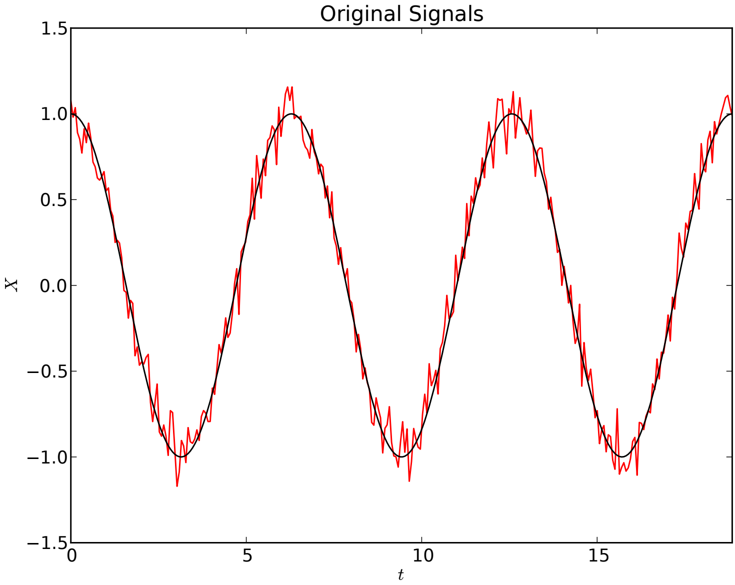


Persistent Homology of Point Cloud





Noise resilience



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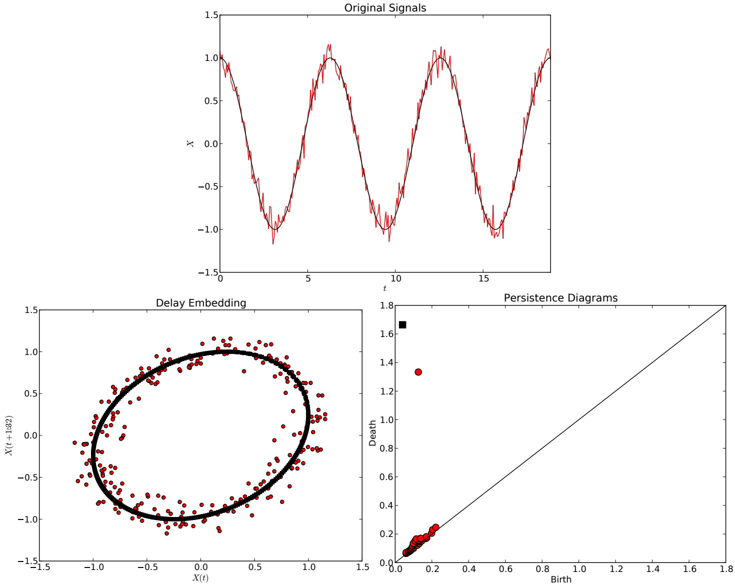
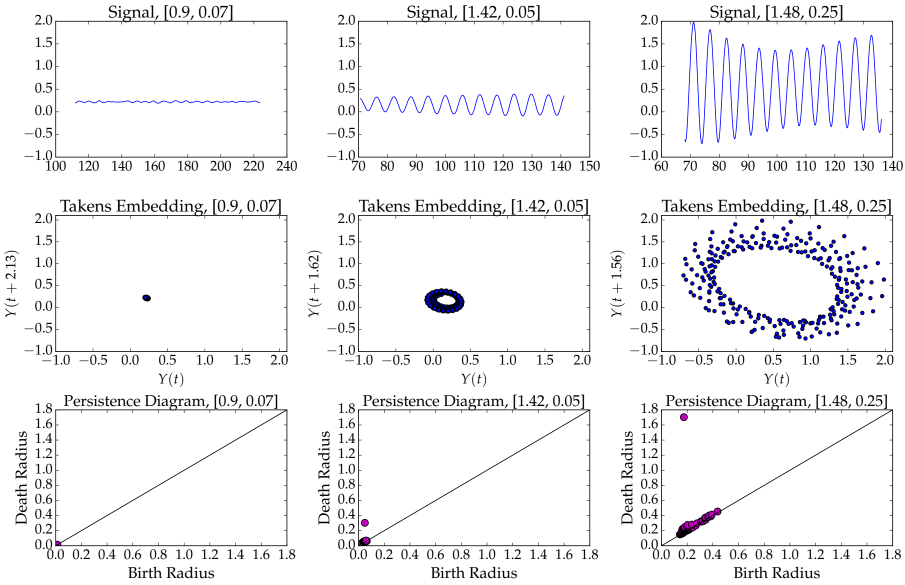


Comparing signals using persistence



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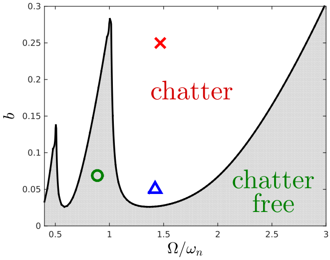
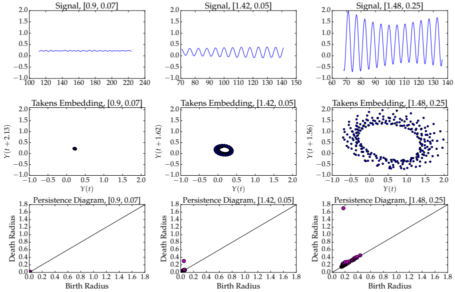
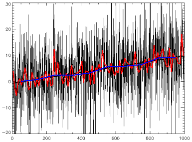
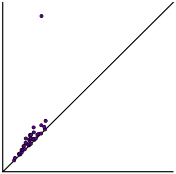




Overview





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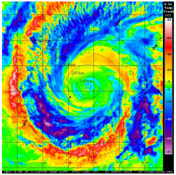

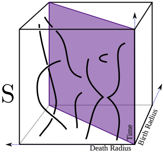

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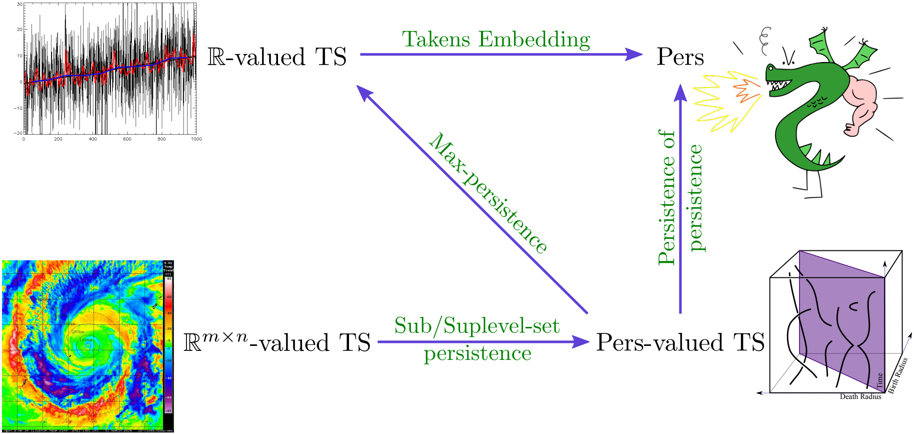
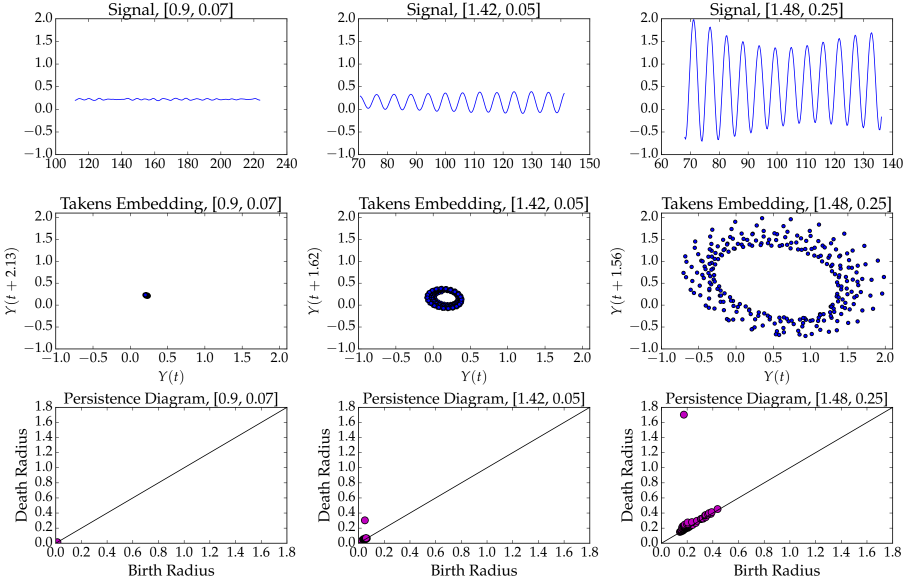
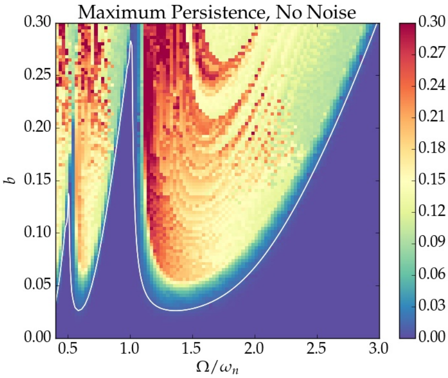
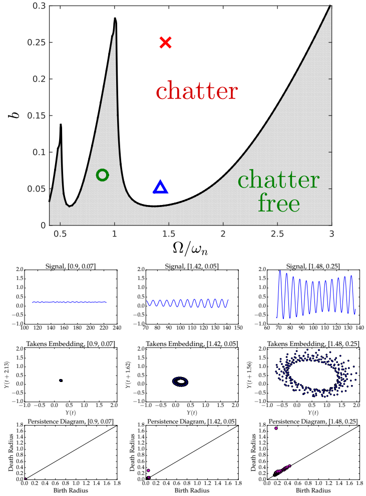

$\mathbb{R}$-valued TS
Takens Embedding
Pers
Max-persistence
Persistence of persistence
$\mathbb{R}^{m\times n}$-valued TS
Sub/Suplevel-set persistence
Pers-valued TS
Time
Birth Radius
Death Radius

Differentiation by Max Persistence

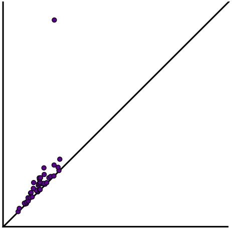


Turning Model





Machine Learning



Adcock et al. Coordinates

Diagrams 0 and 1-dimensional of the form $\{(x_i, y_i)\}$

- $\sum x_i(y_i - x_i)$
- $\sum (y_{max} - y_i)(y_i - x_i)$
- $\sum x_i^2(y_i - x_i)^4$
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- $\max\{(y_i - x_i)\}$

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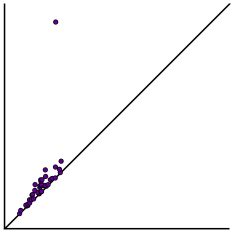

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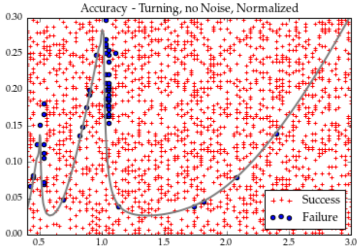
Results (Khasawneh, M, Perea)

- Theoretical stability boundary for training
- Standard logistic classifier
- 97% accuracy



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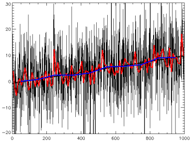
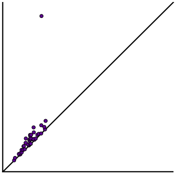
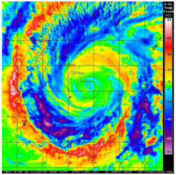


$\mathbb{R}$-valued TS $\xrightarrow{\text{Takens Embedding}}$ Pers

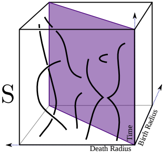


Max-persistence

Persistence of persistence



$\mathbb{R}^{m\times n}$-valued TS $\xrightarrow[\text{persistence}]{\text{Sub/Suplevel-set}}$ Pers-valued TS



1. Classification and Machining Dynamics

2. Periodicity and Hurricanes

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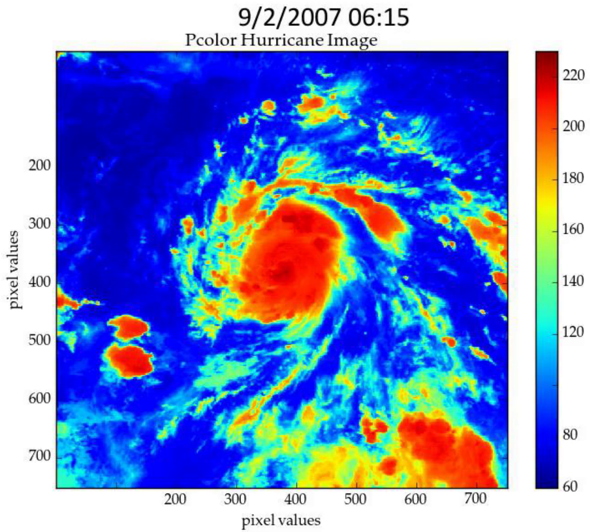
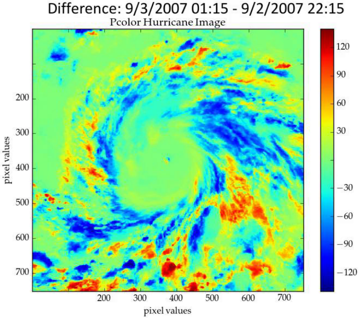
Hurricane Felix



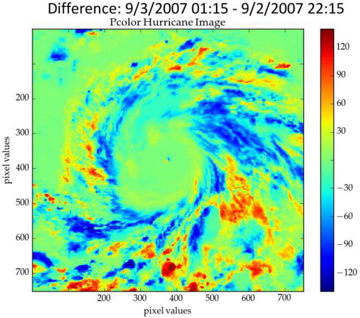
Diurnal cycle



3 hour difference

- $N(t)$ is IR matrix at time t
- $N(t) - N(t - 3 \text{ hrs})$

Diurnal cycle



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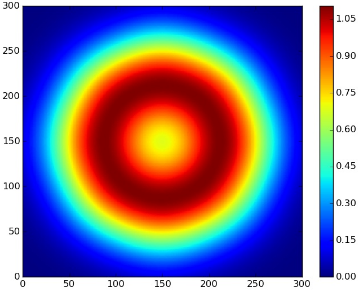
Diurnal cycle

- Sunset: cold ring, "diurnal pulse"
- Starts with radius $\leq$ 150km, spreads outward
- Warm ring forms behind this pulse and spreads outward

Dunion et al. *The Tropical Cyclone Diurnal Cycle of Mature Hurricanes.* Monthly Weather Review, 2014.

Sublevel Set Persistence



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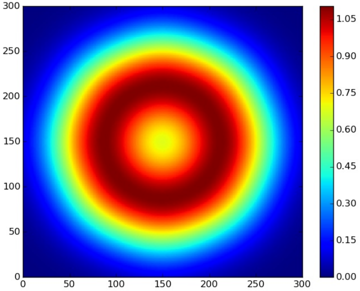

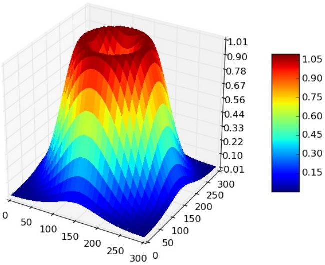

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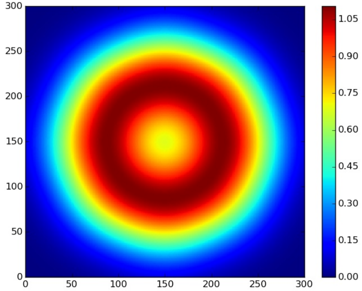

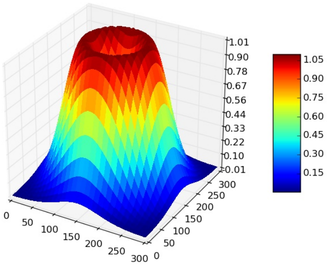

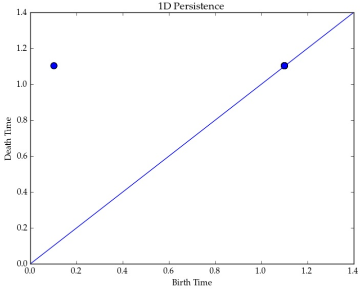

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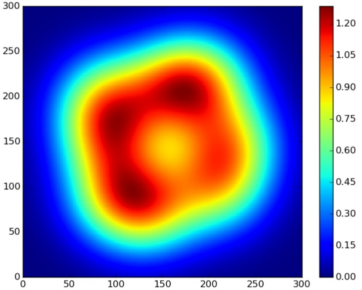
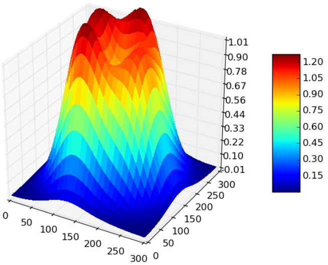
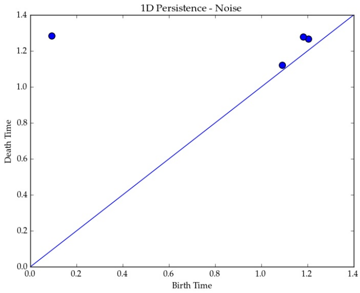
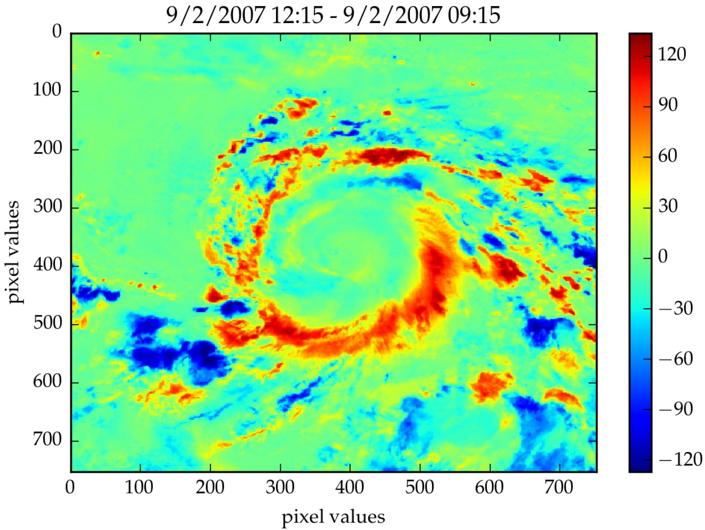


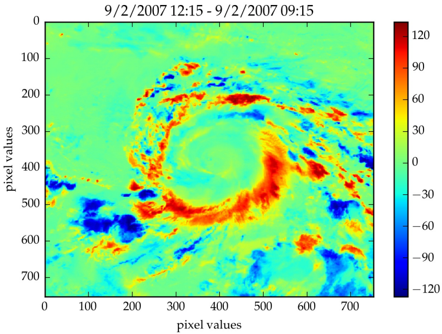




Why the obvious thing doesn't work



Plan B



Definition

Let $K_{m \times n} = K$ be the $m \times n$ grid cubical complex.

Definition

Given $M \in \mathbb{R}^{m \times n}$, let

- $M : K \to \mathbb{R}$
- $M^\mu \subset K$ with function value $\geq \mu$.
- $S : K \to \mathbb{R}$ defined by $S(\sigma) = d(\sigma, M^\mu)$ for $\dim(\sigma) = 2$

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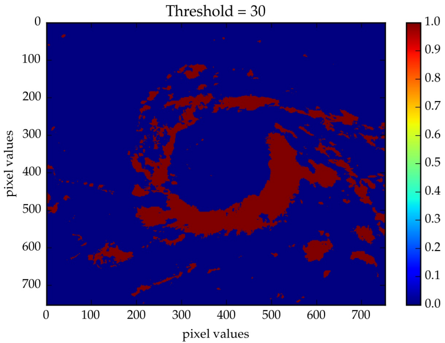

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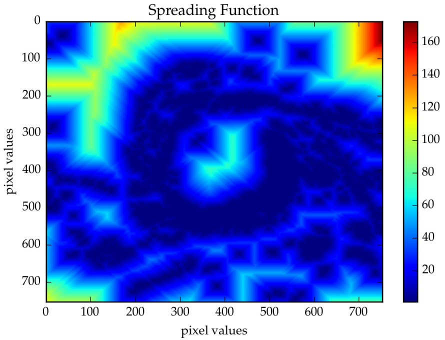

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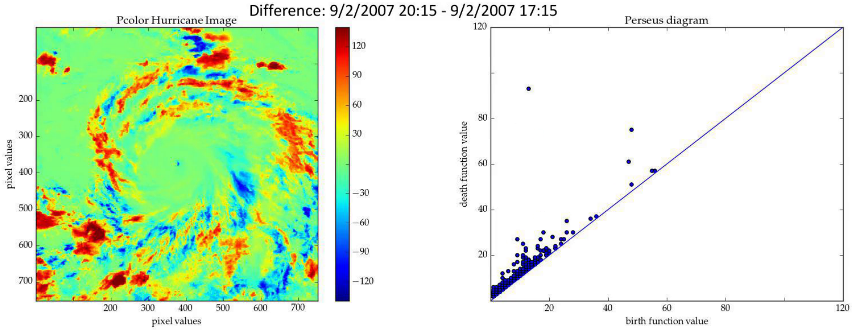
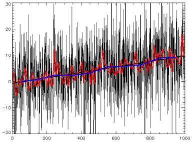
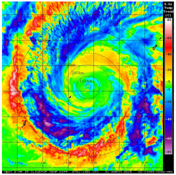
Resulting persistence diagrams

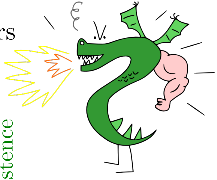


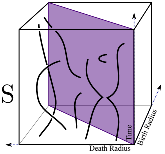
Overview





$$\begin{tikzcd}
\mathbb{R}\text{-valued TS} \arrow[rr, "\text{Takens Embedding}"] & & \text{Pers} \\
\mathbb{R}^{m\times n}\text{-valued TS} \arrow[rr, "\text{Sub/Suplevel-set}", "\text{persistence}"'] & & \text{Pers-valued TS} \arrow[ull, "\text{Max-persistence}"] \arrow[u, "\text{Persistence of persistence}"]
\end{tikzcd}$$





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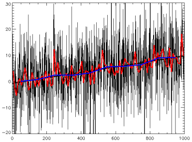

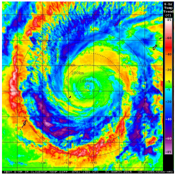

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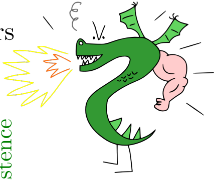
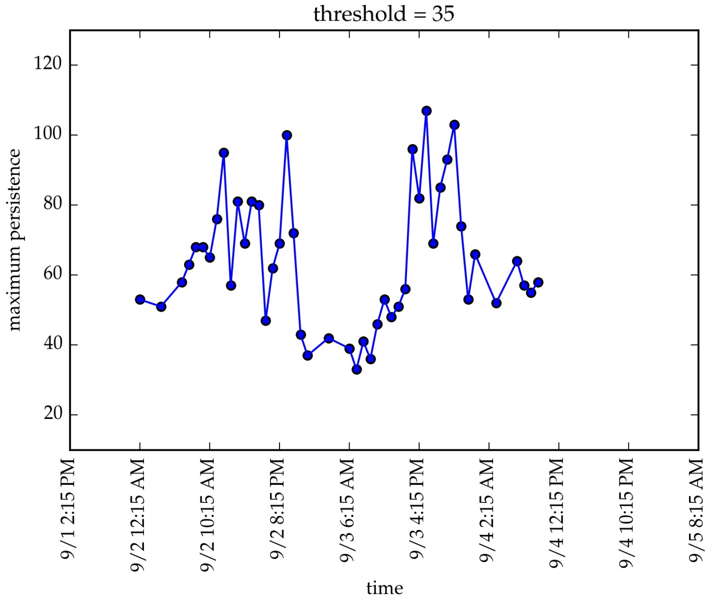
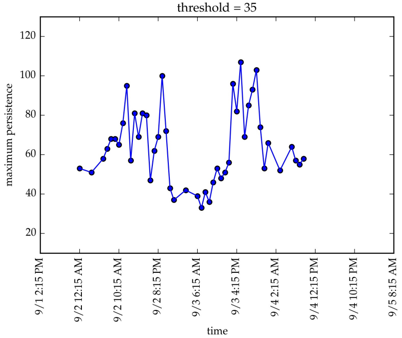
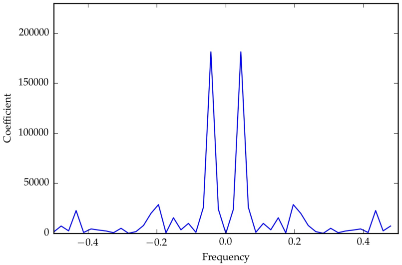


Fourier spectrum of threshold

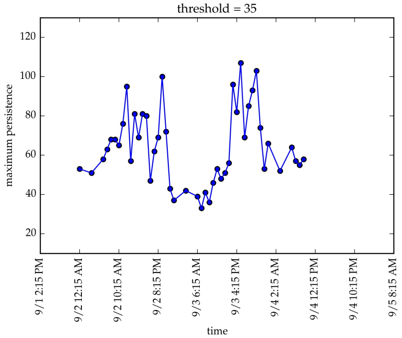


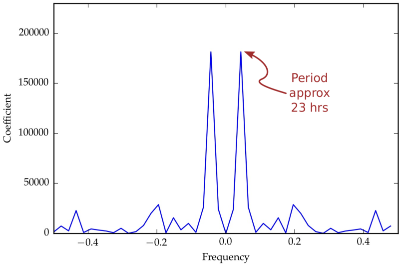
Fourier spectrum of threshold



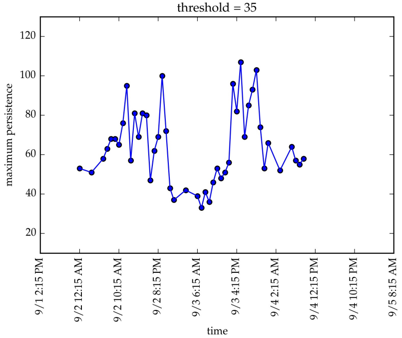


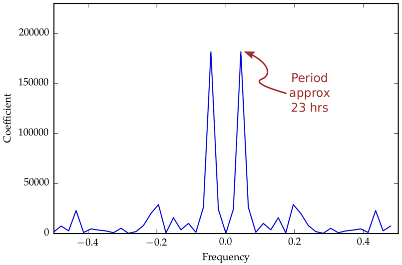
Fourier spectrum of threshold





Fourier spectrum of threshold





Results

- 23 hour day?

General tools for TSA with TDA

- Takens embedding $\rightarrow$ persistence
  - Real-valued time series
  - Can do classification, segmentation using persistence diagrams

General tools for TSA with TDA

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- Image $\to$ sublevelset persistence
  - Get a time series of persistence diagrams
  - Pick out information from each diagram (max pers) to use standard TSA methods
  - Analyze speed
  - Persistence of persistence
    (Kramar, Levanger, et al. 2015 arXiv:1505.06168)

General tools for TSA with TDA

- Takens embedding → persistence
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    (Kramar, Levanger, et al. 2015 arXiv:1505.06168)
- Structures and behaviors that are easy to tease out
  - Circles/holes
  - Periodicty

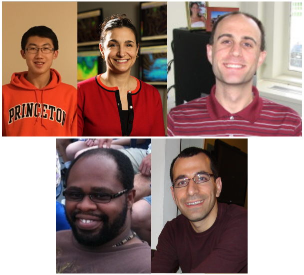
Thank you!

Hurricanes

Kristen Corbosiero (Albany)
Jason Dunion (Albany)
Bill Dong (Guilderland High School)
Ryan Torn (Albany)

Machining Dynamics

Firas Khasawneh (SUNY Poly)
Jose Perea (MSU)



FK and EM. *Chatter detection in turning using persistent homology*. Mechanical Systems and Signals Processing, 2016.

elizabethmunch.com
emunch@albany.edu