Supplemental Material: 2D Vector Field Simplification Based on Robustness

Primoz Skraba*
Jozef Stefan Institute
Slovenia

Bei Wang†
Scientific Computing and Imaging Institute
University of Utah

Guoning Chen‡
University of Houston

Paul Rosen§
Scientific Computing and Imaging Institute
University of Utah

The sampled running times of the various operations used in simplifying the vector fields based on our approach are shown in Table 1.

Dataset Name	Tile #	Region	# of vertices	# of edges	**Reading input** (sec)	**Smoothing** (sec)	**Cut** (sec)
OceanC	20904	(1,6)	120	285	0.007	0.647	0.014
		(8,9)	85	212	0.005	0.521	0.010
Synthetic		SyntheticC	1318	3767	0.100	19.993	0.195
Combustion	173	(10,13)	21662	64035	0.273	166.723	2.007
		(0,3)	1694	4852	0.020	0.1031	0.151
		(5,8)	3949	11474	0.084	0.423	0.542
		(18,20)	1106	3091	0.019	0.070	0.108

Table 1: The running times of the various operations used in simplifying the vector fields. For comparison, we show the time taken to load as inputs the respective vector fields. The code is implemented in MATLAB and the running time is given in seconds. The main bottleneck in the computation is often Laplacian smoothing, whose running time is highly dependent on the chosen parameters. The values shown are obtained with identical parameters to the ones which are used for the results in the paper. Finally we note that in a more recent C++ implementation of the same algorithm, the times have been reduced by at least an order of magnitude and could be further improved with a more optimized implementation.

The simplification algorithm comes with theoretical guarantees on bounding the amount of perturbation we introduce, whenever cutting and/or unwrapping is used. The main motivation for introducing Laplacian smoothing is to produce more visually appealing results. As shown in Figure 1(b), the cutting procedure alone gives a correct, continuous but not visually appealing simplification result, compared to the vector field with Laplacian smoothing in Figure 1(c). To further describe the amount of perturbation we introduce in practice for both our synthetic and real-world datasets, we include Table 2 below. In practice, the addition of Laplacian smoothing does increase the amount of perturbation but not significantly. For most datasets, the total amount of perturbation after **Cut** alone (the 5th column), as well as combining **Cut** and **Smoothing** (the 6th column) is roughly upper-bounded by the robustness of the critical points (e.g. the maximum magnitude of the vector field in the region of interest, the 7th column), as indicated by a radio below 1.

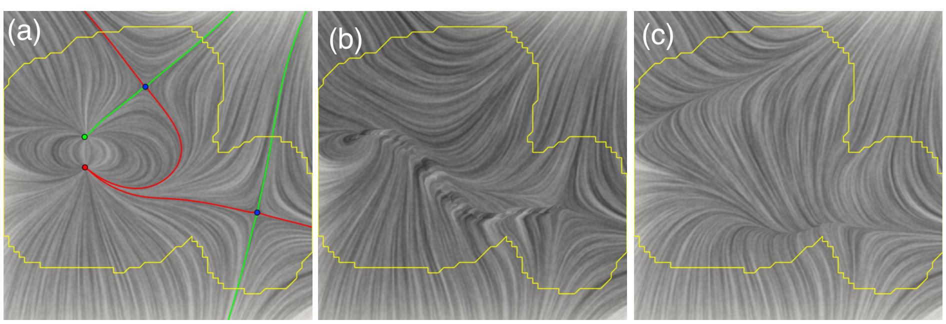

Figure 1: SyntheticB. (a) the original vector field with its topological skeleton. (b) Simplification result by **Cut** only (without smoothing). (c) Simplification result by **Cut** and **Smoothing**.

*e-mail: primoz.skraba@ijs.si
†e-mail: beiwang@sci.utah.edu
‡e-mail: chengu@cs.uh.edu
§e-mail: prosen@sci.utah.edu

Dataset Name	Tile #	Region	**Smoothing**	**Cut**	**Cut** and **Smoothing**	Max Magnitude
OceanA	20311	(6,9)	0.8863	0.4547	0.7162	4.2849
	21217	(8,9)	0.5902	0.5598	0.5727	5.0136
OceanB	20821	(2,4)	0.2869	0.2409	0.2409	8.0435
		(6,7)	—-	0.5432	0.5432	6.3153
OceanC	20904	(1,6)	0.1744	0.3095	0.3095	8.3236
		(8,9)	0.2740	0.5080	0.5080	9.8286
OceanD	20710	(4,5)	0.7971	0.5296	0.6801	8.3474
		(7,8)	0.3533	0.1923	0.2830	6.7341
	20715	(5,7)	1.2277	0.7006	0.9239	8.0544
		(8,9)	—-	0.5040	0.5040	10.7337
Synthetic		SyntheticA	1.0744	1.1701	1.0744	0.0059
		SyntheticB	1.2003	1.1706	1.2003	0.0059
		SyntheticC	1.4375	1.1706	1.2024	0.0059
Combustion	173	(3,6)	0.1490	0.0936	0.1490	0.1220
		(10,14)	—-	0.5944	0.6016	0.2542
		(12,13)	0.4614	0.2674	0.2699	0.0699
		(18,19)	—-	0.3626	0.3669	0.1998
		(10,13)	0.2775	0.9446	0.9448	0.4557
		(0,3)	—-	0.1439	0.1439	0.2583
		(5,8)	—-	0.4559	0.4559	0.3015
		(18,20)	0.0846	0.2661	0.2661	0.1109

Table 2: Amount of perturbation introduced during our simplification algorithm. The first three columns indicates the specific regions of interest, where the 3rd column includes the coordinates of the regions to be simplified. For a particular region C, the amount of perturbation (or vector field distortion) introduced by the simplification is shown as a ratio with respect to the radius of $\mathrm{im}(C)$ (which is approximately the robustness value), which is given by the maximum magnitude of the region C (7th column). The 4th column includes distortion introduced using just Laplacian **Smoothing**. The 5th column is for **Cut** procedure (possibly preceded by **Unwrap**) and the 6th column is for **Cut** and **Smoothing** (corresponds to results shown in the paper). For the synthetic datasets, for **Cut** operation only, the values could be brought arbitrarily close to 1 through an appropriate choice of ε (see Section 4.3). Some values are missing for **Smoothing** as these operations do not remove the critical points.