

RECURSION REVISITED
THE RISE OF GOOGLE

SEARCHING AND SORTING

ANNOUNCEMENT

- Bonus Project 3 Posted: Essay
- Please go to TA's office hours: this should be a norm, not an anomaly
- Quiz 3 posted: have 1 week to talk to the TA and request in writing regrading
- Class participation: 5%. TAs are keeping track...you are already here, why not actively participate?

RECURSION REVISITED

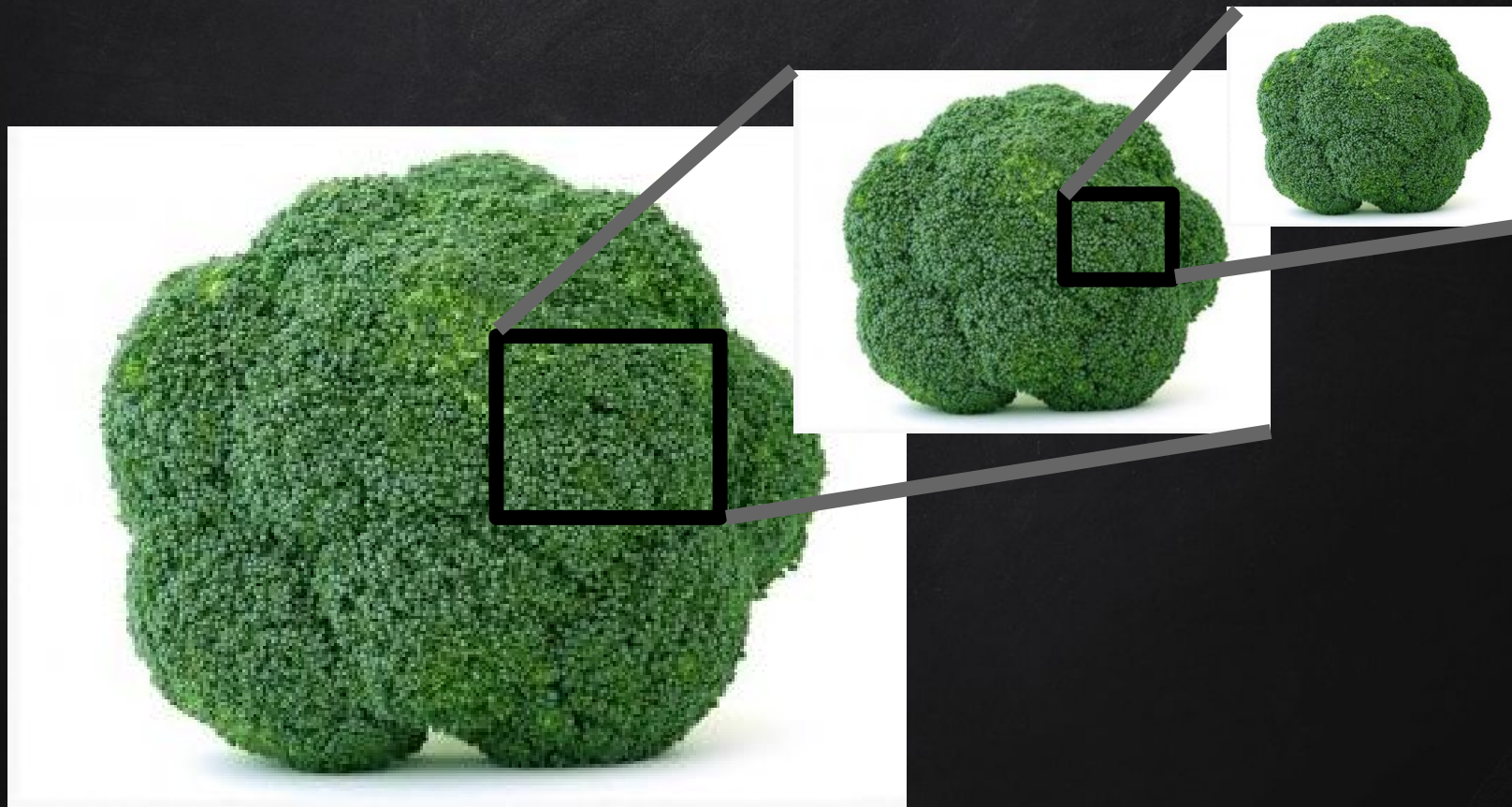
<http://www.pythontutor.com/index.html>

WHAT IS RECURSION?

A picture of a painter who is painting a picture of painter who is painting a picture ...



A Broccoli



WHAT IS A RECURSION?

- Joke: in order to understand recursion, you need to understand recursion...
- A recursion function is a function that calls itself
- Recursion is hard to understand...some people get it, some don't

Two things:

1. Understand how to solve a simpler problem
2. Understand how to trace a recursive function

THE HATE-LOVE-HATE RELATIONSHIP WITH RECURSION

1. You hate it because you do not understand it
2. You love it because it is cool after you understand it
3. You hate it because it is typically inefficient



CLASSIC RECURSION

Solving a "big" problem recursively means to solve one or more smaller versions of the problem, and using those solutions of the smaller problems to solve the "big" problem.

BIG-VERSION V.S. SMALL-VERSION OF THE SAME PROBLEM

- Solving problems recursively typically means that there are smaller versions of the problem solved in similar ways.
- Think about summing over an array of 10 numbers v.s. summing over an array of 5 numbers.
- Use the same technique: a counter
- Solution to the smaller problem helps you to solve the larger problem.

BASIC UNIT OF A RECURSION IS A FUNCTION CALL

Four steps to understand recursion

1. Write and define the prototype of the function
2. Write out a sample function call
3. Think of the smallest version of the problem
4. Think of smaller version of the function call

Putting it all together!

TASK: SUM UP NUMBERS
FROM 1 TO X

1. WRITE AND DEFINE THE PROTOTYPE OF THE FUNCTION

```
def compute_sum(x): # add numbers from 1 to x
```

2. WRITE OUT A SAMPLE FUNCTION CALL

```
def compute_sum(x):    # add numbers from 1 to x
```

```
...
```

```
print compute_sum(1)
```

```
print compute_sum(2)
```

```
print compute_sum(3)
```

3. THINK OF THE SMALLEST VERSION OF THE PROBLEM

Base case: the smallest version of the problem

Base case here: $x = 1$

Base case is where the recursion eventually stops

When $x = 1$, `compute_sum(x)` should return 1

```
def compute_sum(x):
```

```
    if x == 1:
```

```
        return 1
```

<https://www.cs.umd.edu/class/fall2002/cmsc214/Tutorial/recursion.html>

4. THINK OF SMALLER VERSION OF THE FUNCTION CALL

`compute_sum(x)` # compute sum from 1 to x

`compute_sum(x-1)` # compute sum from 1 to x-1

If we want to solve a bigger problem with solving a smaller problem first:

`compute_sum(x) = x + compute_sum(x-1)`

PUTTING IT ALL TOGETHER!

```
def compute_sum(x):  
    if x == 1:  
        return 1  
    else:  
        return x + compute_sum(x-1)
```

PUTTING IT ALL TOGETHER!

```
def a_simple_recursive_function(x):  
    if (base case)  
        return some simple expression  
    else:  
        some work before  
        recursive call  
        some work after
```

```
1- def f(x):
2-     if x == 1:
3-         return 1
4-     else:
5-         return (x + f(x-1))
6
7 print f(1)
8 print f(2)
9 print f(3)
10 print f(4)
11 print f(5)
```

1
3
6
10
15

CLASSIC RECURSION

Thinking “backwards”:

Instead of building a solution from nothing, you pretend you are at the solution, and want to take a step back and ask how to solve the problem if you were a step back.

Alternatively, **thinking about how the solution of a bigger problem can be constructed from a solution of a smaller problem.**

`compute_sum(x) = x + compute_sum(x-1)`

ANOTHER EXAMPLE

How to reverse a string?

`reverse_string('hello')` → "olleh"

`reverse_string("abcdefg")` → "gfedcba"

`reverse_string("abcdefg")`: put "g" first, add it to the reversed result of "bcdefg"

`reverse_string(str) = str[l-1] + reverse_string(str[0:l-1])`

ANOTHER EXAMPLE

```
def a_simple_recursive_function(x):  
    if (base case)  
        return some simple expression  
    else:  
        some work before  
        recursive call  
        some work after
```



```
1- def f(mystr):
2-     if len(mystr) == 1:
3-         return mystr
4-     else:
5-         l = len(mystr)
6-         return (mystr[l-1] + f(mystr[0:l-1]))
7
8- print f('hello')
9- print f('a')
10- print f('abcde')
11- print f('123456')
12- print f('[]')
```

```
olleh  
a  
edcba  
654321  
][
```

FINAL EXAMPLE

Compute $a*b$ with only additions/subtractions:

$a*b = b + b + \dots b = a$ copies of b

1. multiply(a,b)
2. multiply(3,4)
3. multiply(1, b)
4. multiply(a,b)
 - a. multiply(a-1, b)
 - b. multiply(a,b) = b + multiply(a-1,b)

```
1- def multiply(a,b):  
2-     if a == 1:  
3-           
4-     else:  
5-           
6-  
7- print multiply(2,4)  
8- print multiply(9,100)  
9- print multiply(1,10)  
10-
```

```
8  
900  
10
```

```
1- def multiply(a,b):
2-     if a == 1:
3-         return b
4-     else:
5-         return b + multiply(a-1, b)
6
7- print multiply(2,4)
8- print multiply(9,100)
9- print multiply(1,10)
```

MORE READINGS ON RECURSION

Tracing recursive functions:

<http://www.pythontutor.com/index.html>

<https://www.cs.umd.edu/class/fall2002/cmsc214/Tutorial/trace-recursion.html>

More readings:

<https://www.cs.umd.edu/class/fall2002/cmsc214/Tutorial/recursion.html>

Try tracing the following code in <http://www.pythontutor.com/index.html>

Using “Forward” step by step

```
def f(x):  
    if x==1:  
        return 1  
    else:  
        return x + f(x-1)  
  
print f(3)
```

SEARCHING LISTS

DATA STRUCTURES

An important part of computer science is data structure:

- A data structure is a particular way of storing data so it can be processed efficiently

For example: Storing numbers

A TYPE OF DATA STRUCTURE: LISTS

Imagine tracking the names of basketball players who scored in a basketball game:

Brekott Chapman, Isaiah Wright, Austin Eastman, Jake Connor, Brandon Taylor, Dakarai Tucker, Lorenzo Bonam

Chris Reyes, Jordan Loveridgel, Kenneth Ogbe, Gabe Bealer, Jayce Johnson, Kyle Kuzma, Makol Mawien, Austin Montgomery

Jakob Poeltl

- Need to store that information in a variable
- Make a new variable for each scorer: scorer1, scorer2, ...
- Use as needed as the game progresses



LISTS

A list is a collection of information

- Variable length
- Can add or remove item from the list
- Can look at items in a list

What are some examples of lists in real-life?

LISTS IN REAL LIFE

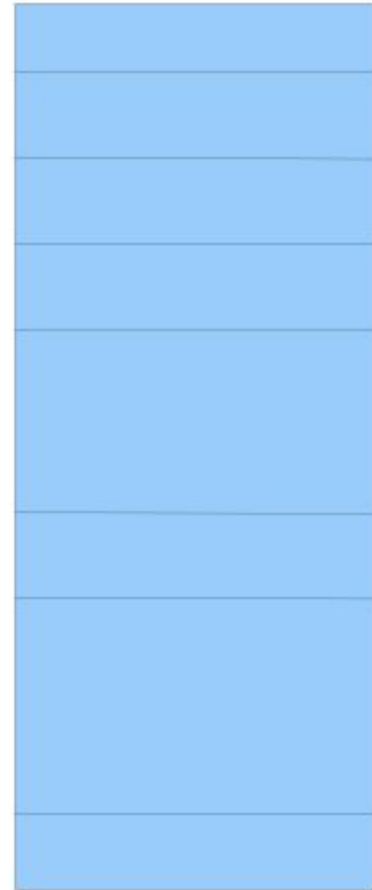
Shopping list:

```
shopping_list = ['apple', 'orange', 'meat', 'napkin']
```

LISTS ON A COMPUTER

- You can make lists different ways on a computer
- One way is to use an array: in Python-- it is a list
- Each individual item is accessed by its place in the collection
- We call the location number for a value an **index**

length



[0]

[1]

[2]

[3]

[length-1]

[MAX_LENGTH-1]

MORE LISTS IN PYTHON

```
shopping_list = ['apple', 'orange', 'meat', 'napkin']
```

```
course_list = ['physics', 'chemistry', 'computer science']
```

```
number_list = [1, 2, 3, 4, 5]
```

```
alphabet_list = ['a', 'b', 'c', 'd']
```

REVIEW: LISTS

Lists

- Contain multiple items
- Expands to hold as many items as needed
- Look at a particular item with an index number
- Add, remove, replace items in a list


```
course_list = ['physics', 'chemistry', 'english', 'biology'];  
number_list = [1, 2, 3, 4, 5, 6, 7];
```

```
print "course_list[0]: ", course_list[0]  
print "number_list[1:5]: ", number_list[1:5]
```

```
course_list[0]: physics  
number_list[1:5]: [2, 3, 4, 5]
```

```
course_list = ['physics', 'chemistry', 'english', 'biology'];
```

```
print "Value at index 2 :"
```

```
print course_list[2]
```

```
course_list[2] = 2001;
```

```
print "New value at index 2 :"
```

```
print course_list[2]
```

```
print course_list
```

Value at index 2 :

english

New value at index 2 :

2001

['physics', 'chemistry', 2001, 'biology']

```
course_list = ['physics', 'chemistry', 'english', 'biology'];
```

```
print "Value at index 2 : "
```

```
print course_list[2]
```

```
print course_list
```

```
del course_list[2]
```

```
print "After deleting value at index 2 : "
```

```
print course_list
```

Value at index 2 :

english

['physics', 'chemistry', 'english', 'biology']

After deleting value at index 2 :

['physics', 'chemistry', 'biology']

```
course_list = ['physics', 'chemistry', 'english', 'biology'];
```

```
print course_list[2]
```

```
print course_list[-1]
```

```
print course_list[1:]
```

english

biology

['chemistry', 'english', 'biology']


```
course_list = ['physics', 'chemistry', 'english', 'biology'];
```

```
print len(course_list)
```

```
print [1,2,3]+[4,5,6]
```

```
print 'hello'*4
```

```
print 3 in [1,2,3]
```

```
for x in course_list:
```

```
    print x
```

4

[1, 2, 3, 4, 5, 6]

hellohellohellohello

True

physics

chemistry

english

biology

COMING UP NEXT:
MORE ON LISTS
AND SORTING



THANKS!

Any questions?

You can find me at
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<http://www.sci.utah.edu/~beiwang/teaching/cs1060.html>

CREDITS

Special thanks to all the people who made and released these awesome resources for free:

- Presentation template by [SlidesCarnival](#)
- Photographs by [Unsplash](#)