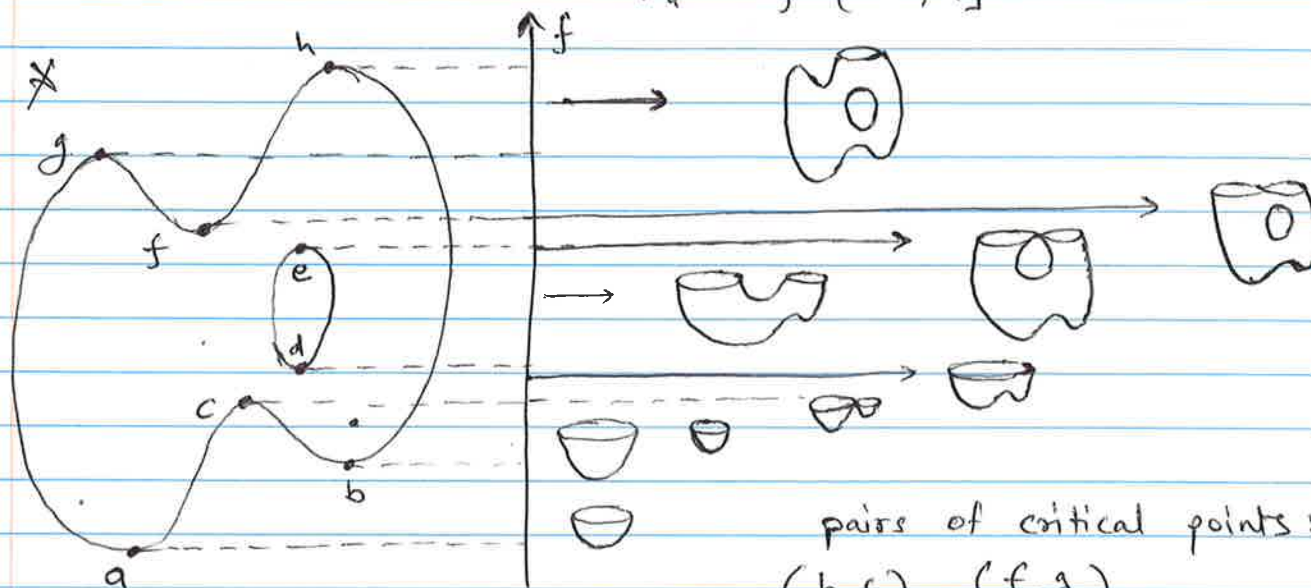
Apr 4

Extended Persistence

Consider $f : \mathbb{X} \to \mathbb{R}$ and the sub level sets. $X_a = f^{-1}(-\infty, a]$

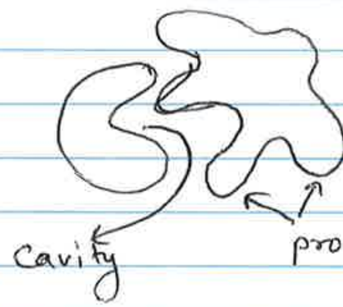
Reeb Graph



pairs of critical points: (b,c) (f,g)

Extended persistence pair (a,h), (d,e) → Essential features.
These are the features created during sub-level set filtration but are not destroyed (eg. the tunnel)

Protein Docking!

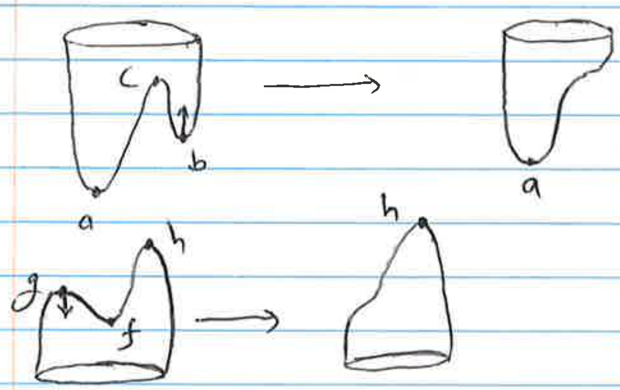


Given two complementary shapes, we want to find best way to "dock" them.

→ Protrusion / Cavity can be described by the persistence of persistence pairs.

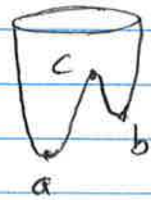
which protrusion best fits the cavity.

eg. $(b,c) \rightarrow f(c) - f(b) =$ persistence gives the "size" of the cavity / protrusion.



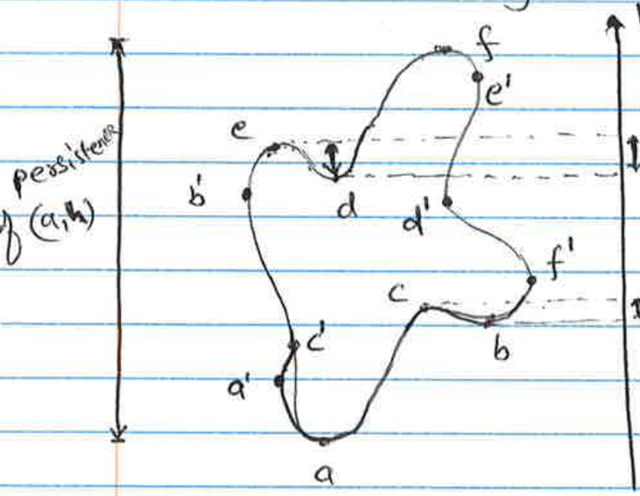
persistence simplification.
introduce perturbation which gets rid of the protrusion
amount of perturbation is equal to persistence of pair (b,c)

persistence simplification described earlier is equivalent to eliminating branches b-c and f-g from Reeb graph.



→ simplification can be done by ① raising b ② lowering c ③ both ① & ② simultaneously

raising b → both ← lowering c

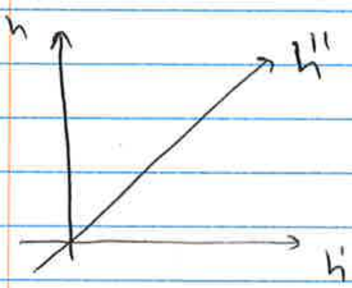


persistence of (d,e)

persistence of (b,c)

→ pairs : ~~(xxxx)~~ (b,c) (d,e) (a,f) along direction of h.

→ h' : pairs : (b', c'), (d', e'), (a', f') along h' direction.

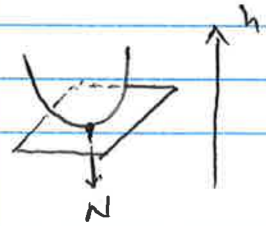


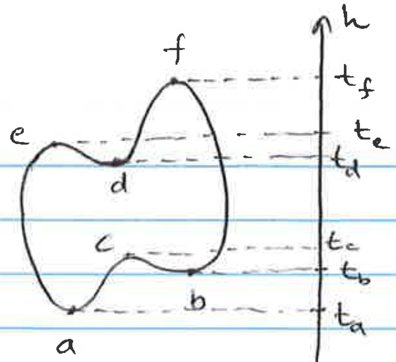
along each direction, local minima, local maxima, saddles (critical points) are different, giving different persistence pairs.

Elevation function : $E : \mathbb{X} \to \mathbb{R}$.

for $x \in \mathbb{X}$, $E(x)$: persistence of x when it becomes critical

→ point x becomes critical ⟹ normal direction at x aligns with the "height" directions





$$E(b) = E(c) = |t_c - t_b|$$
$$E(d) = E(e) = |t_e - t_d|$$
$$E(a) = E(f) = |t_f - t_a|$$

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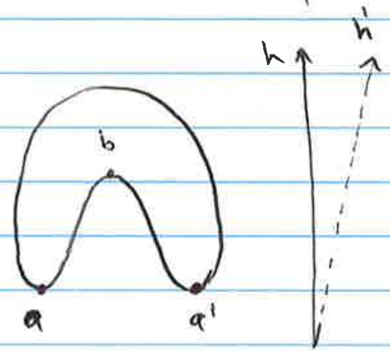

in case of function h, both a & a' have same "height". Both can be paired with b.
if we slightly perturb h to say h' then a' is no longer global minima so the pairing is no longer ambiguous.

2 - legged case

This is a degenerate case.

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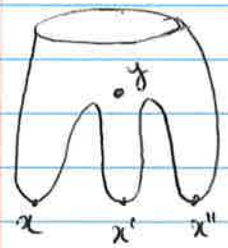
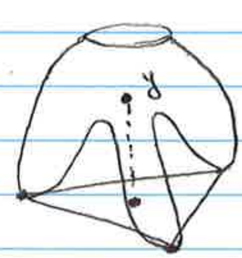


local saddle y, all three local minima, x, x', x'' have same height, any one point can be paired with y.

Three - legged case



→ projecting y down along height function direction the projection falls inside the triangle formed by x, x', x''.

#

→ two saddles y, y' and two local minima, x, x'
projecting y-y' ridge down shows a crossing with line joining x - x'

4 - legged case