CS 6170 Computational Topology: Topological Data Analysis — **University of Utah**
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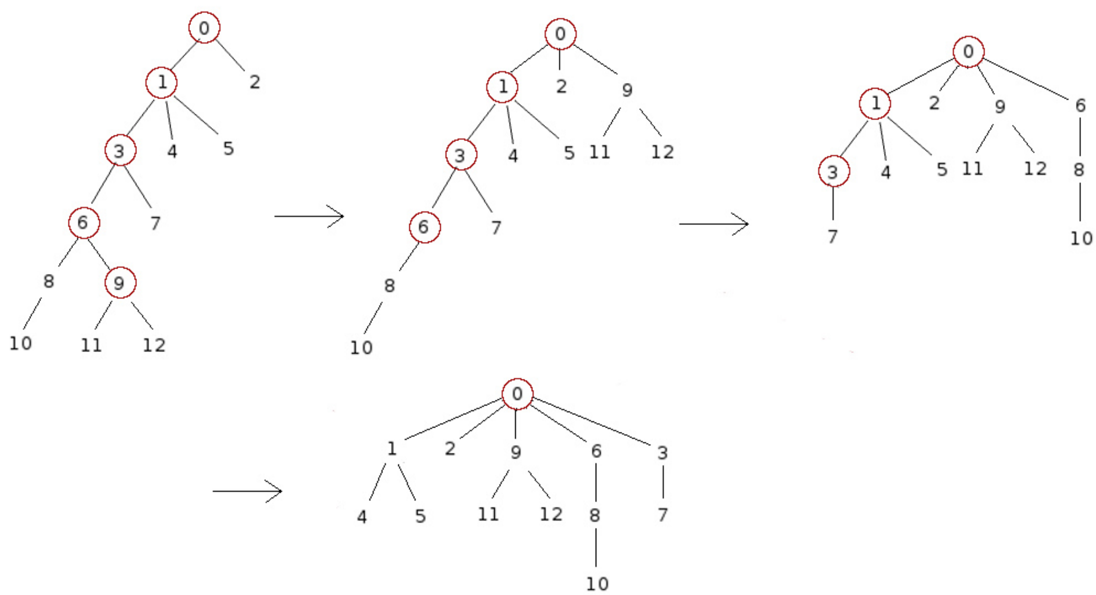
4.1 Union Find

4.1.1 Path compression example

Path compression steps for $Find(9)$ algorithm.



4.1.2 Pseudo-code

MakeSet(x)
  $parent(x) \leftarrow x$
  $rank(x) \leftarrow 0$

Find(x)
  if $x \neq parent(x)$ **then**
    $parent(x) \leftarrow Find(parent(x))$
  return $parent(x)$

Union(x, y)
  $A \leftarrow Find(x)$
  $B \leftarrow Find(y)$
  if $rank(A) > rank(B)$ **then**
    $parent(B) \leftarrow A$
  else $parent(A) \leftarrow B$
  if $rank(A) = rank(B)$ **then**
    $rank(B) = rank(B) + 1$

Claim: $rank(x)$ is upper bound of the true rank of x.

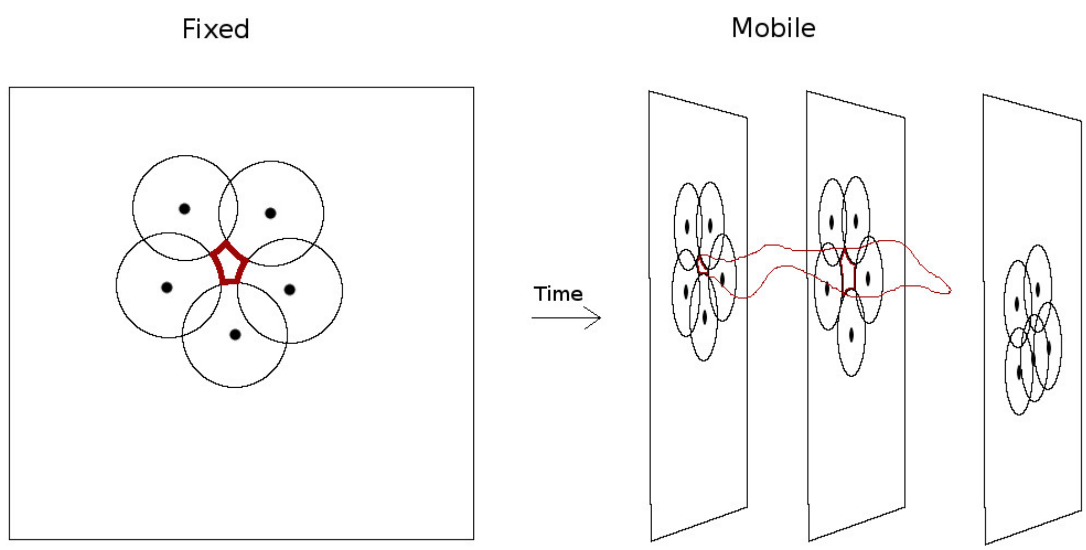
4.2 Sensor Network Coverage (Data Motivation)

4.2.1 Examples

Consider a room containing robots that are able to detect an intruder within a certain radius. Also consider what happens when the robots are fixed and when they are able to move.

Problem: How to determine sensing coverage of the room.

Some topological techniques can be used to determine coverage. In the image labelled 'fixed' the red portion represents a tunnel. However, in the 'mobile' representation the red is not a tunnel by the third frame.



The robots can modelled as vertices. We put an edge between the two vertices if their coverage areas overlap. These graphs can be used to find *simplicial complexes*.

Simplicial complex examples:

- Cech complex - form a d-simplex when there is a common point of intersection of all the $\frac{\epsilon}{2}$-balls.
- Rips complex - form a d-simplex if there are pairwise intersections among all the $\frac{\epsilon}{2}$-balls.

In the robot example the d-simplexes will be 2-simplexes which are triangles. A rips complex is easier to compute; however, there may not be a point of intersection for all $\frac{\epsilon}{2}$-balls (i.e. a hole). There will be no holes in Cech complexe, but it is harder to compute.

The School of Computing story "To Catch a Wireless Theif"[U2016] gives another example of sensor network coverage. Sneha Kasera created a system to determine the location of someone stealing bandwidth. The system uses volunteer devices to detect a thief via an app. This system makes a intruder detection type of sensory network similar to the robot example. The goal of this project is to use crowd-sourcing to develop a network large enough to monitor all frequency at all times in all areas.

4.3 Simplex

Definition: Given $k+1$ points in $\mathbb{R}^d$ $U_0, u_1, ...u_k$ They are affinely independent iff k vectors $u_i - u_0 (1 \leq i \leq k)$ are linearly independent.

Definition: A convex hull of a set of points X in $\mathbb{R}^d$ is the smallest convex set that contains X. (Think of a rubber band stretched around pegs.)

Definition: A point $X = \sum_{i=0}^{k} \lambda_i U_I (\lambda_i \in \mathbb{R})$ is an affine combination of U_i if $\sum_{i=0}^{k} \lambda_i = 1$. Further, X is a convex combination if all $\lambda_i \geq 0$.

Definition: A k-simplex σ is the convex hull of $k+1$ affinely independent points. $\sigma = Conv\{u_0, u_1, ...u_k\}$ dimension of σ, $dim(\sigma) = k$.

$k = 1$	0-simplex	a point
$k = 2$	1-simplex	a line segment
$k = 3$	2-simplex	a triangle
$k = 4$	3-simplex	a tetrahedron
$k = 5$...	4-simplex...	other polytopes

References

[U2016] To Catch A Wireless Thief. (2016, July 19). *UNews* Retrieved from unews.utah.edu