Applied topology in visualization

Between beautiful concepts and practical needs

Guest Lecture, Salt Lake City, February 7 2019

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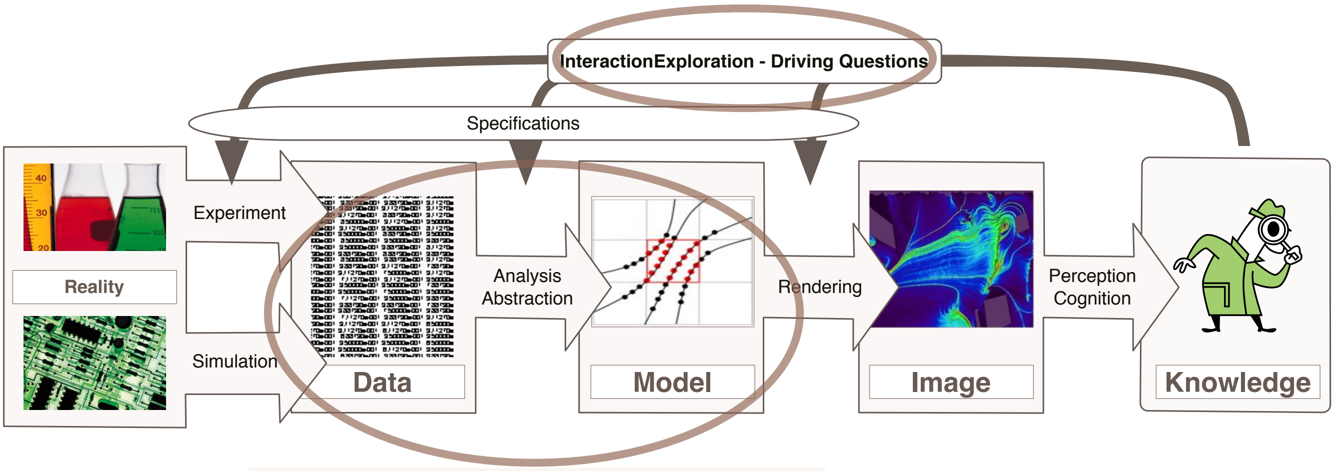
Scientific Visualization, Linköping University

Visual Data Analysis

Specific to questions



Complexity reduction
Concepts – Techniques – Algorithms

Visual Data Analysis

Generate an environment for scientific reasoning through visual interaction with data

- Methods for data reduction and abstraction tailored to specific **questions**
- **Topology is one way to approach this goal**

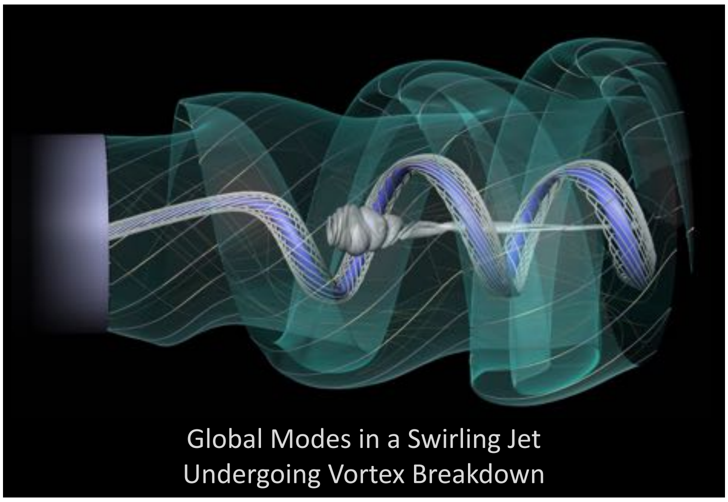
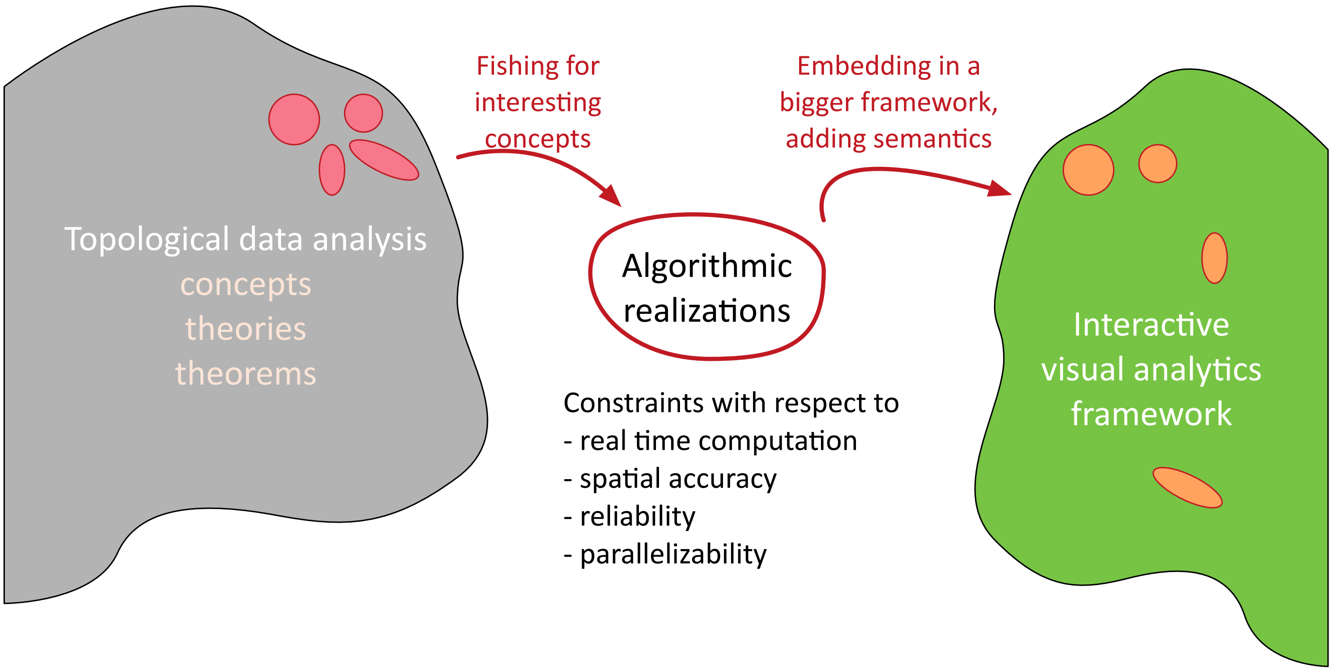


Image: Petz, ZIB, Amira

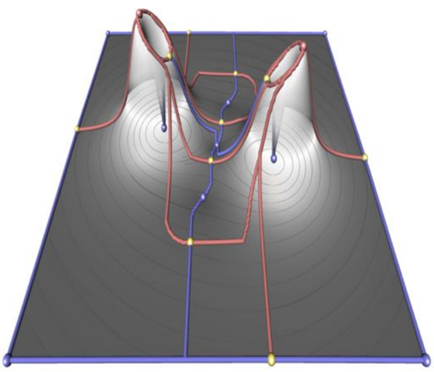
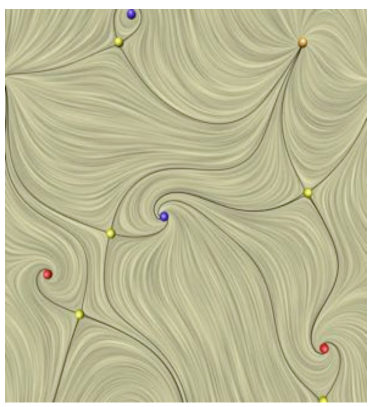
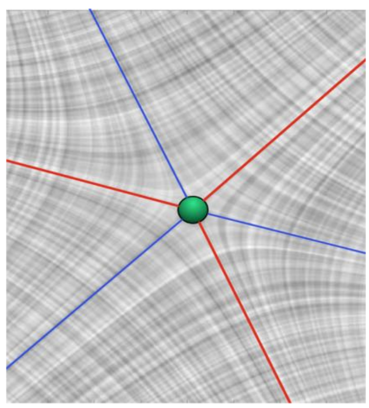
Topology in Visualization

Topology in visualization



Topological features in visualization – historical development

Scalar fields	Vector fields	Tensor fields
Critical points Maxima, Minima, Saddles Separatrices	**Limit sets** Sources, Sinks, Saddles Closed Orbits **Separatrices**	**Degenerate points** Trisector, Wedgepoints Separatrices

Vector Field Topology in Visualization

Vector field topology – Basic ingredient

Streamlines (Integral curve)

- Everywhere tangential to vector field at fixed time

Let $v\colon \mathrm{D} \rightarrow \mathbb{R}^3$ be a vector field

A **streamline** of ***v*** at **time *t₀*** is a curve

$$c : I \rightarrow D$$
$$s \mapsto c(s)$$

parameterized by $s \in I = [0, S] \subset \mathbb{R}$

and
$$c(0) = \mathbf{x}_0$$
$$\frac{\mathrm{d}c}{\mathrm{d}s} \parallel \mathbf{v}(c(s), t_0)$$

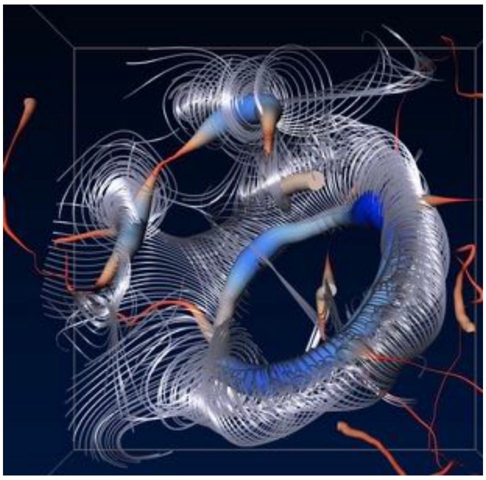


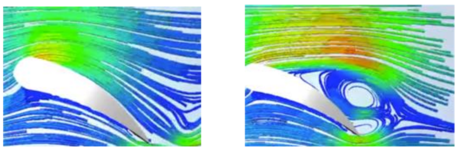
Image: Jens Kasten, ZIB, Amira

Vector field topology – Motivation mostly comes from flow analysis

Typical Questions

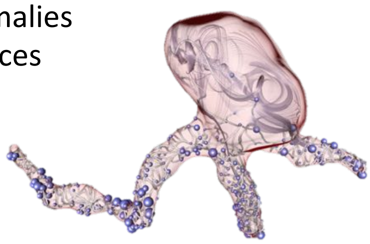
Flow around a body (e.g. car, airplane)

- Vortex formation
- Flow separation



Medicine – flow in blood vessels

- Anomalies
- Vortices



Vortex in blood flow in aneurysm

Combustion and fuel injection into engines

Pollution distribution of particles in the atmosphere or water systems

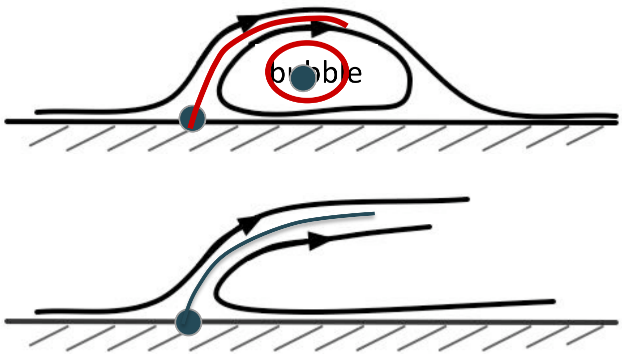
→ Mixing process

Mixing of a fluid – color pH value of fluid.
CAP Arts of Physics, vis thymol blue.

Vector field topology – Motivation mostly comes from flow analysis

Anticipated typical flow structures

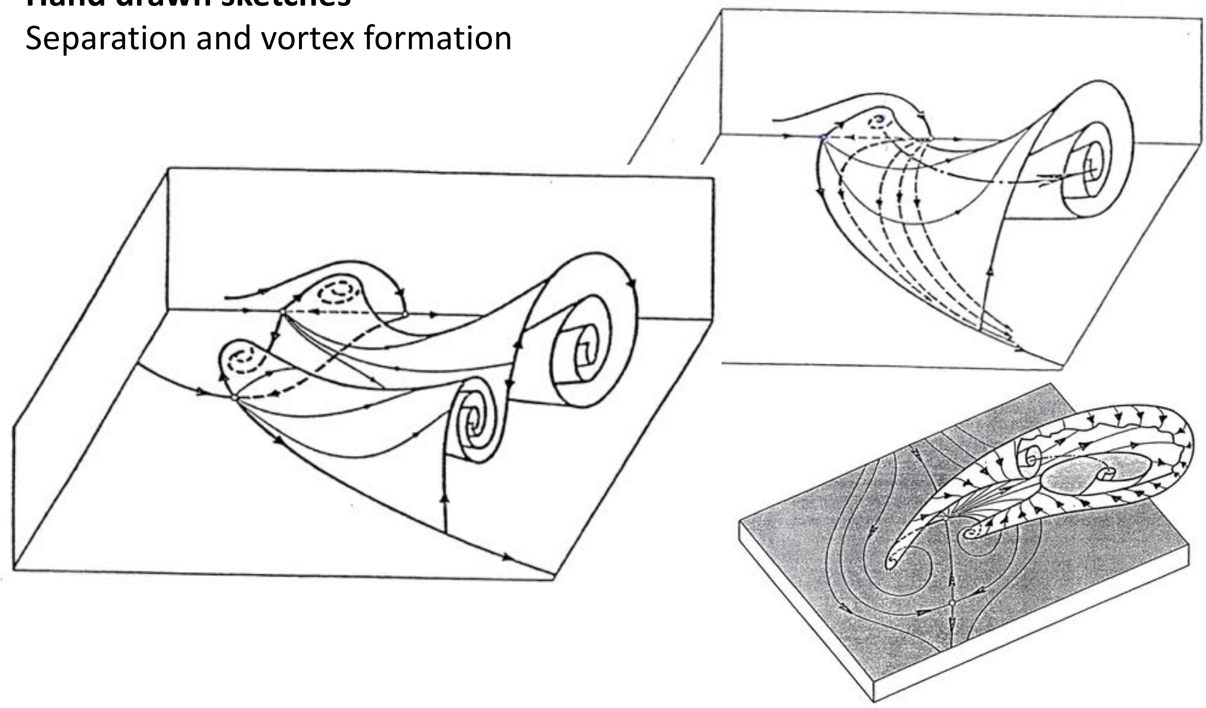
- Relation of vortex formation and separation?
- Characteristic singularities of the flow field?



Often recirculation zones form behind obstacles .
Does separation cause recirculation?

Vector field topology – Motivation mostly comes from flow analysis

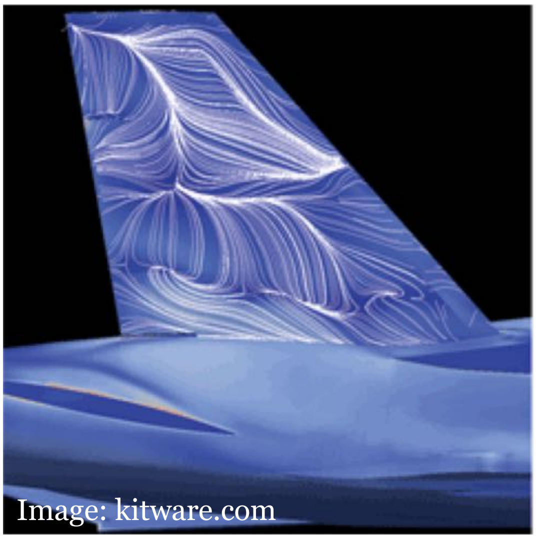
Hand drawn sketches
Separation and vortex formation



Images: Dallmann, German Aero Space, DLR

Vector field topology – Motivation mostly comes from flow analysis

Obviously there is some structure in most vector field data.
Feature extractions tries to make this structure explicit.

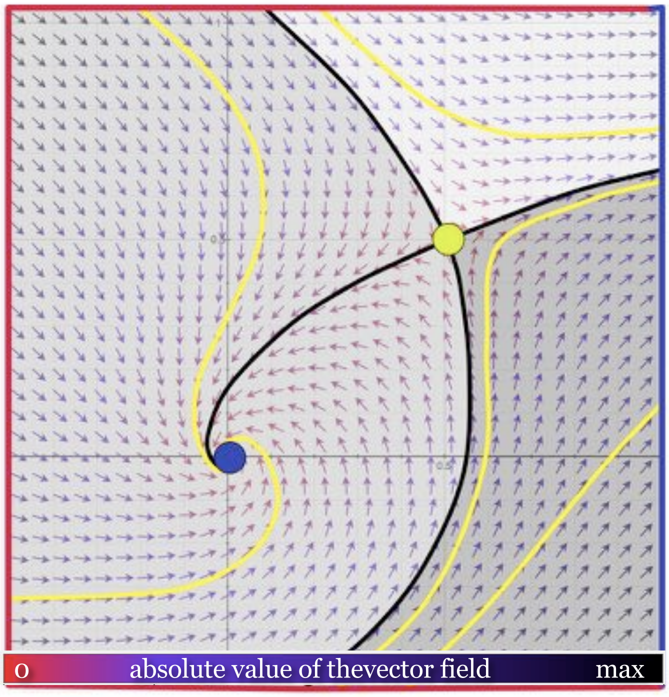


Vector field topology – Intuition

A few streamlines

What about the other streamlines?
Can we tell where they go?

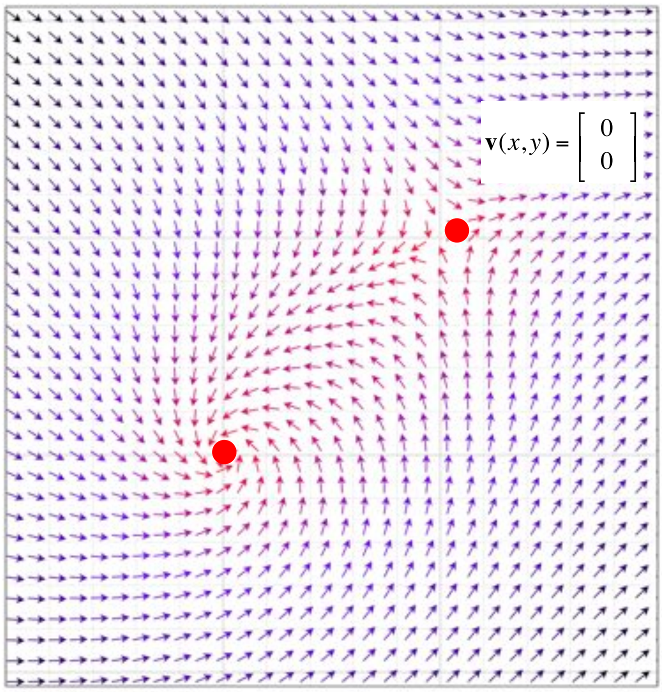
→ Vector field topology answers this question for **ALL** streamlines.



Vector field topology – Intuition

Ingredients

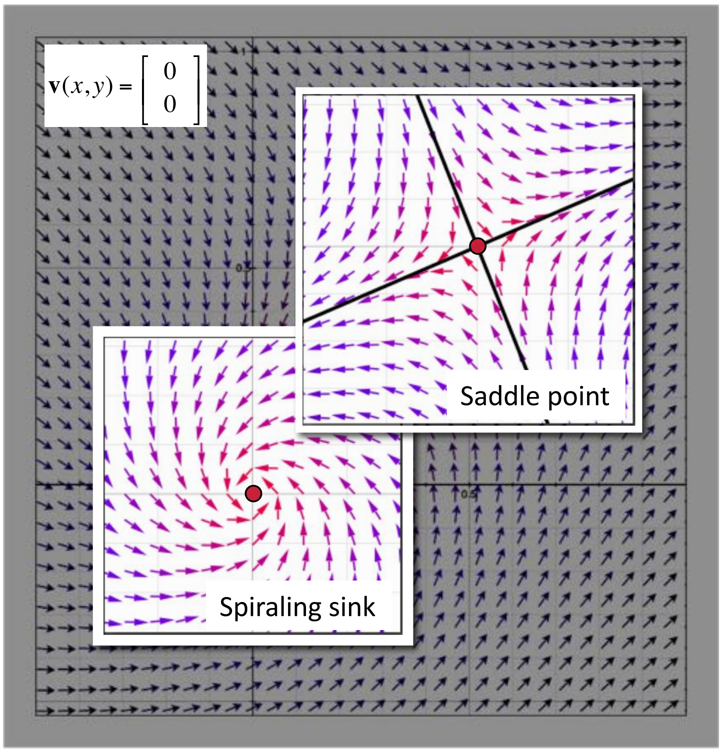
1. Critical points – zeros
   – Positions



Vector field topology – Intuition

Ingredients

1. Critical points – zeros
   - Positions
   - Classification



Vector field topology – Intuition

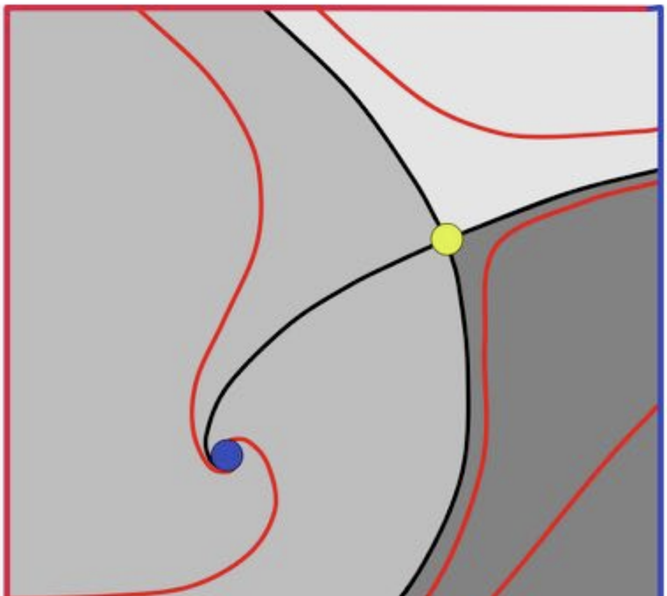
Ingredients

1. Critical points – zeros
   - Positions
   - Classification
2. Separatices

→ Segmentation of domain into areas of similar streamline behavior

Based on ideas from Poincaré over qualitative investigations of differential equations (19th century) ,

Theory of dynamical systems



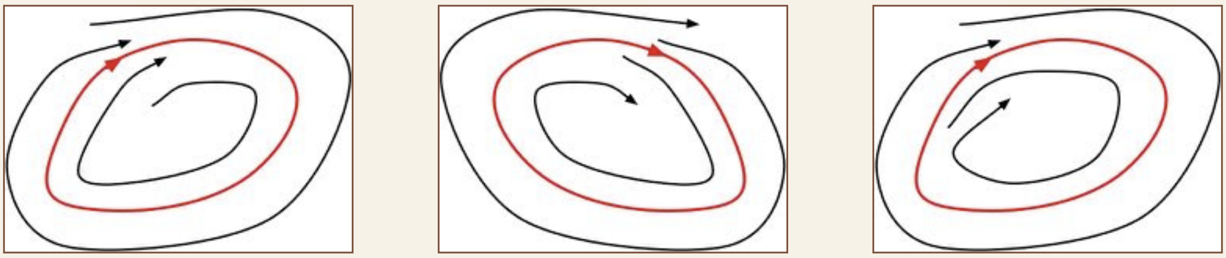
Segment characterized by
Streamline with "same origin" and "same end"

Vector field topology – Basic concept **LIMIT SETS**

Critical points: Zeros of the vector field (Local definition)

Alternative terms: singularities, singular points, zeros, stagnation points

Closed orbits: attracting or repelling (No local definition)



Extracting closed streamlines robustly is a challenging task

There are also boundary contributions

Vector field topology – Basic concept **LIMIT SETS**

Streamline origin / destination

→ Define **start-set / end-set** for every streamline

Definition

α-limit (ω-limit) set to streamline c_p through point P

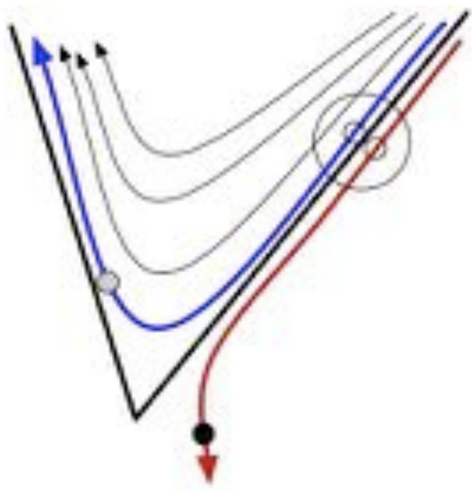
for vector field $\mathbf{v} : D \rightarrow \mathbb{R}^n$

$$\mathrm{A}(c_p) := \left\{ q \in D \mid \exists (t_n)_{n=0}^{\infty} \subset R \text{ with } \lim_{n\to\infty} t_n = -\infty, \text{such that } \lim_{n\to\infty} c_p(t_n) = q \right\}$$

$$\Omega(c_p) := \left\{ q \in D \mid \exists (t_n)_{n=0}^{\infty} \subset R \text{ with } \lim_{n\to\infty} t_n = \infty, \text{ such that } \lim_{n\to\infty} c_p(t_n) = q \right\}$$

Separatrices

- Limiting curves – Separatrices are streamlines connecting the saddle points with other critical points



Vector field topology – Basic concept

→ **The topological graph or skeleton** of a planar 2D vector field consists of all limit sets and separatrices

Vector field topology – Classification

Given a linear vector field
$v: D \rightarrow \mathbb{R}^3$ with $v(x) = A \cdot x + b$,
where $A \in \mathbb{R}^{3\times3}$ and $b \in \mathbb{R}^3$

The matrix **A** can be used to classify the behavior of the vector field in the neighborhood a critical point.

Linear vector fields

- More complex vector fields can be first order approximated by linear vector fields (use Jacobi-Matrix).
- Linear vector fields can be analyzed relatively easily
- On tetrahedral grids with linear interpolation we deal with linear fields

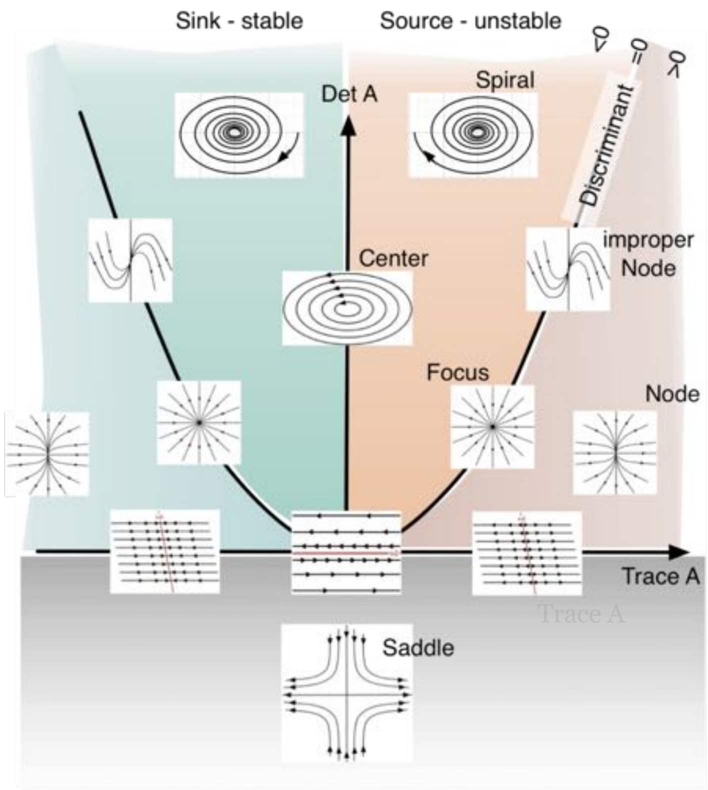
Vector field topology – Linear vector fields

Classification of critical points based on eigenvalues of A

$$\lambda_{1/2} = \frac{\text{tr}(\mathbf{A})}{2} \pm \sqrt{\Delta}$$

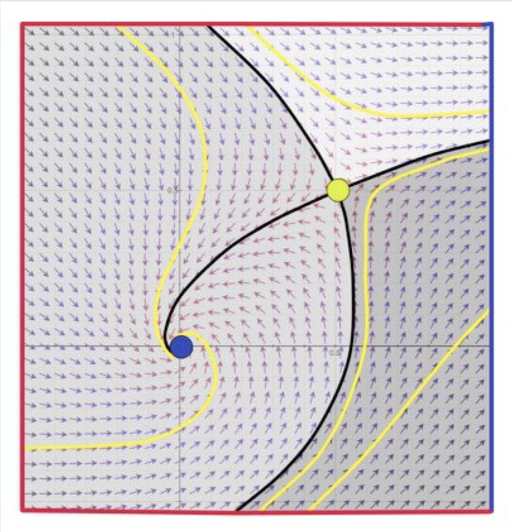
$$\underbrace{\frac{1}{4}\text{tr}^2(\mathbf{A}) - \det \mathbf{A}}_{\text{Discriminant } \Delta}$$

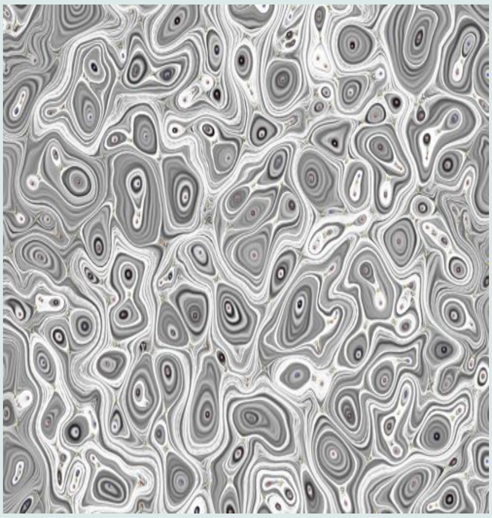
$$tr\ A = a_{11} + a_{22} + a_{33}$$
$$\det A = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$$



Vector field topology – Challenges

Topological graph segments the domain into equivalence classes of streamlines
-- coherent limit behavior



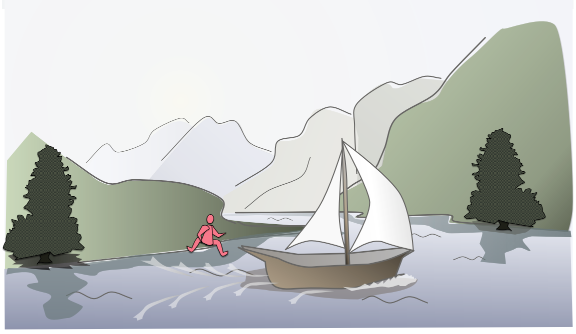


Vector field topology – Challenges

- Often flow data is time-dependent
- The concept of limit set loses its meaning for data given for limited time interval
- Critical points become dependent on the frame of reference

Eulerian view

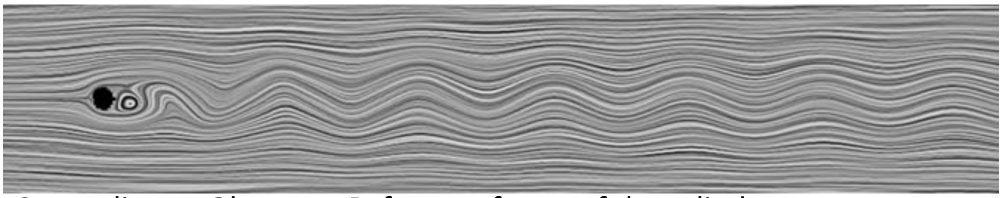
- Observer has fixed Position
- No individual particles are considered
- Position, velocity, … are associated with grid ***v****(x)*



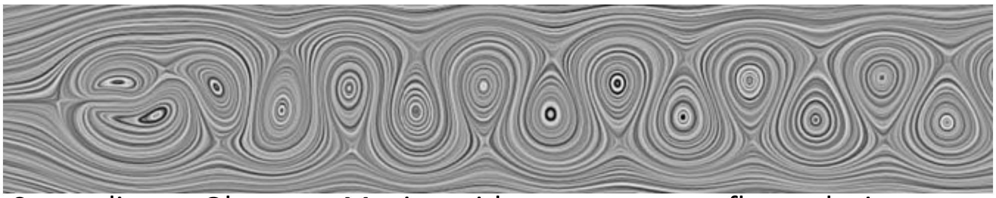
Lagrangian view

- Observer moves with particles
- ‘Individual particles’ can be identified
- Position, velocity, … are associated with particle *i*: $\mathbf{v}_i(t)$

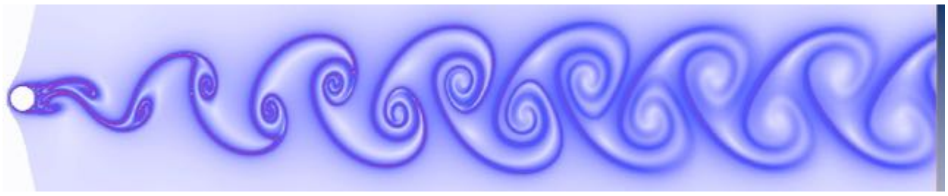
Vector field topology – Looking for coherent structures



Streamlines – Observer: Reference frame of the cylinder



Streamlines – Observer: Moving with constant mean flow velocity



Lagrangian perspective (FTLE): Highlighting separation of particles
FTLE: finite time Lyapunov exponent

Kasten et al. Localized Finite-time Lyapunov Exponent for Unsteady Flow Analysis. 2009

Two Vector field topology application

Uniform streamline placement

Vector field topology for streamline placement

Typical placements

- Interactive choice of single start points
- Start streamlines in all mesh vertices
- Start streamlines at random positions

→ Often very inhomogeneous coverage

Goals

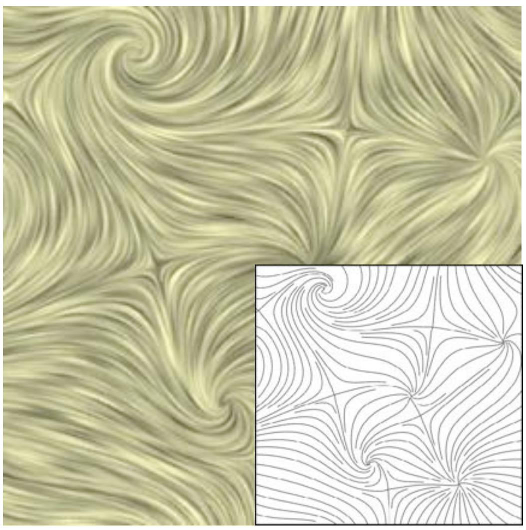
- Coverage
- Uniformity
- Continuity
- Highlight features (CPs)

Olufemi Rosanwo et al. *Dual Streamline Seeding*. 2009

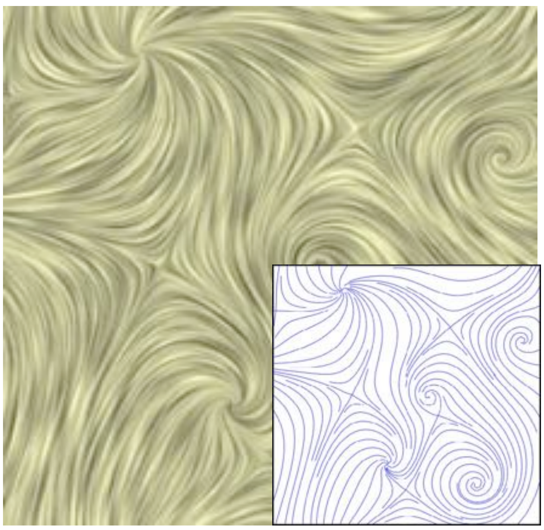
Vector field topology for streamline placement

Idea: Use dual vector field as auxiliary structure

Input: Primal field **v**



Dual field R(**v**) = **v** × n



Images: Rosanwo, ZIB

Vector field topology for streamline placement

Topologies of both fields serves as initialization

→ Both fields have identical critical points

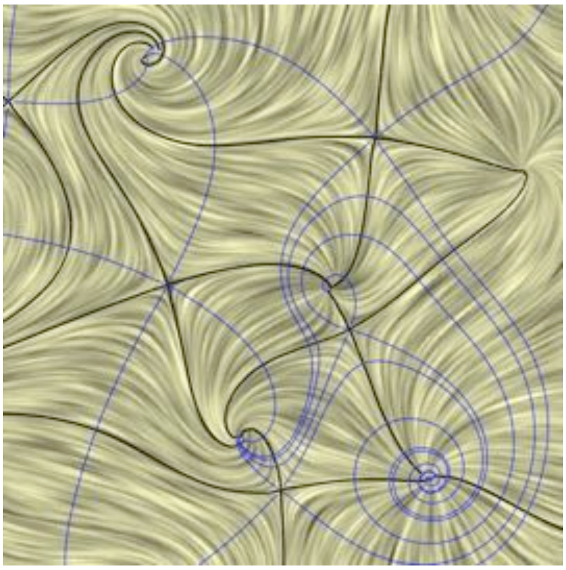
Dual critical points

→ Saddles - saddles (rotated)

→ Spirals – spirals (inverse rotation)

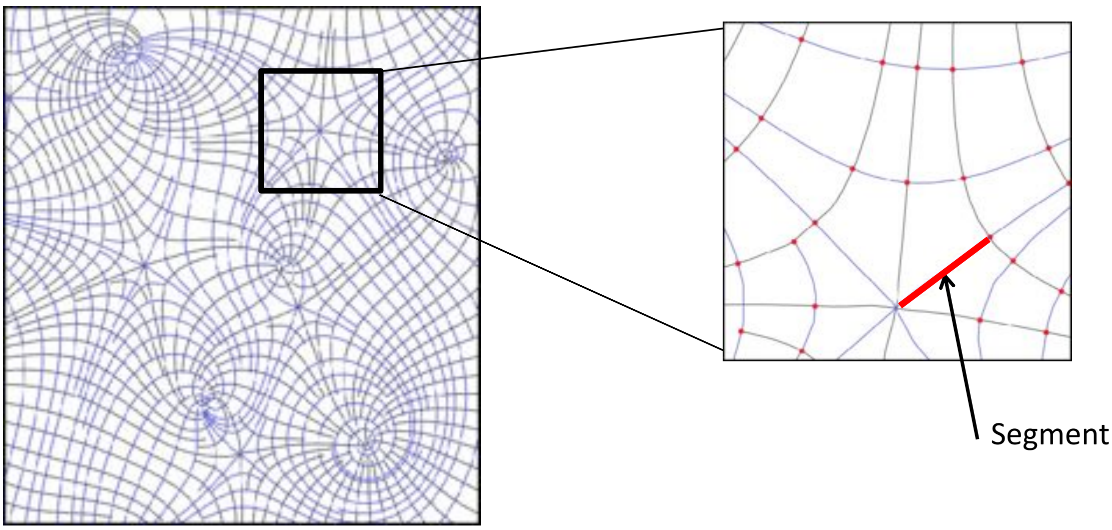
→ Center - focus

Result: Quadrangular cells of varying size



Vector field topology for streamline placement

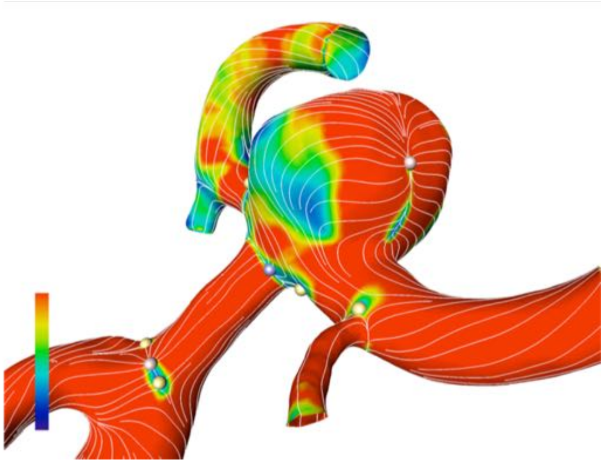
- Streamline seeding in the center of long segments
- Only streamlines of the primal field are shown in the final image



Vector field topology for streamline placement

Blood flow analysis in aneurysms for treatment planning

- **Motivation:** Rupturing aneurysms lead often to the death of the patient.
- **Data:** Blood flow simulation based on imaging data.
- **Goal:** Flow simulations and visualizations shell help to predict the of the rupture risk.

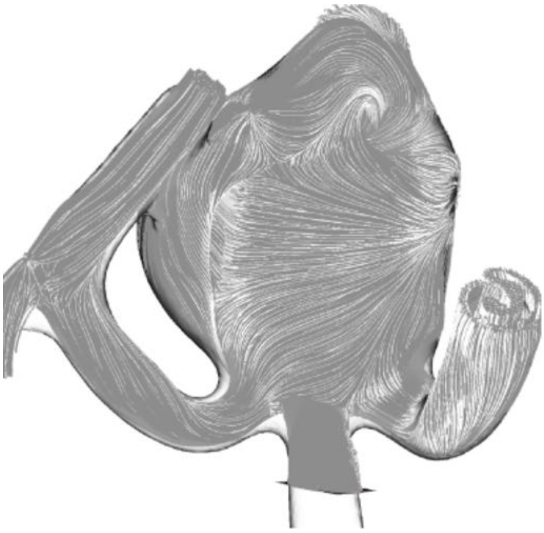


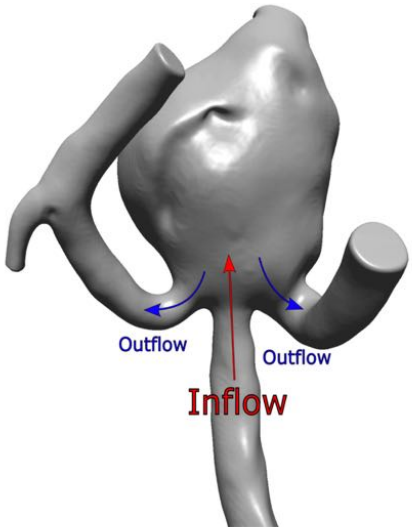
Visualization of the wall shear stress and flow stagnation points on the aneurysm wall.

Two Vector field topology application

Coherent flow structures for blood flow clustering

Vector field topology – Looking for coherent structures

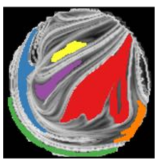


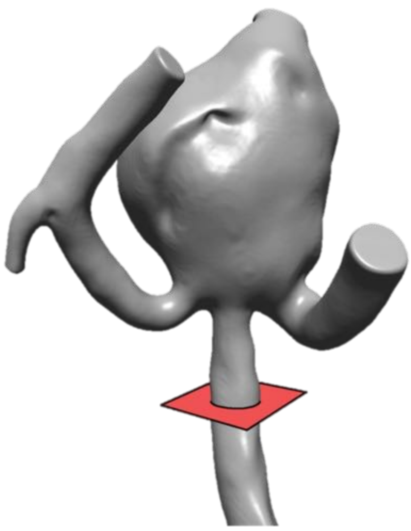


Data: Leonid Goubergrits
German Heartcnter. Berlin

Rickard Englund et al. *Coherence Maps for Blood Flow Exploration*. 2017

Coherence maps for flow clustering



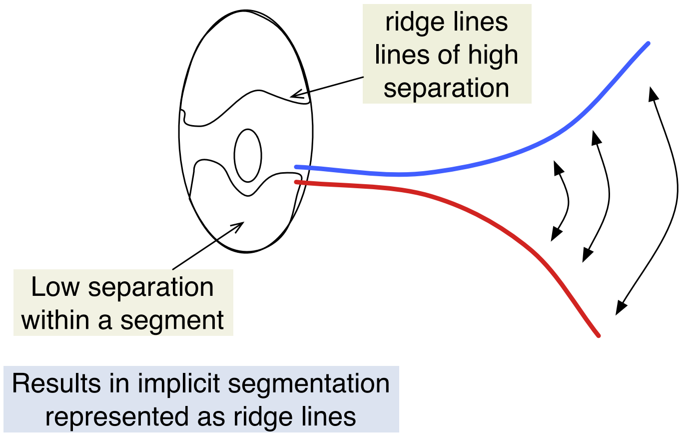


Data: Leonid Goubergrits
German Heartcnter. Berlin

Visualization: Rickard Englund

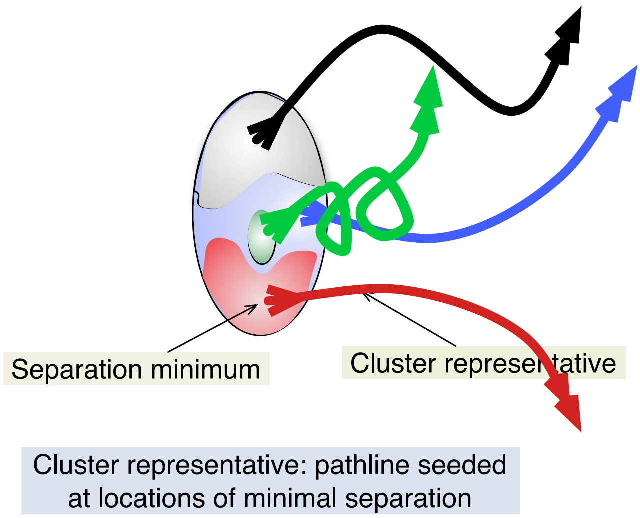
Coherence maps for flow clustering

Finite time Lyapunov Exponent
FTLE – measures separation and coherence



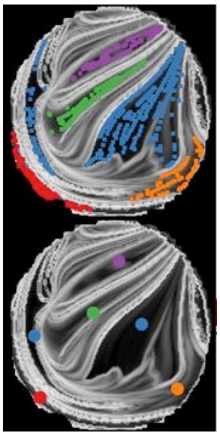
Coherence maps for flow clustering

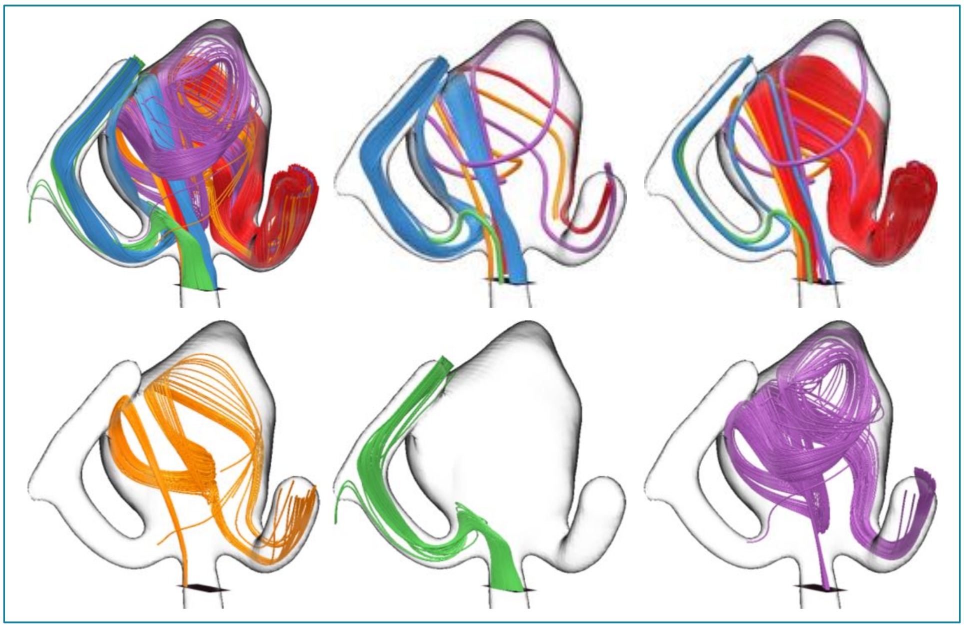
FTLE – measuring separation and coherence



Coherence maps for flow clustering

- Summary of the prominent behavior of the flow in terms of similar flow patterns.
- The map serves as interface for interactive exploration and the derivation of key measures of the individual clusters.

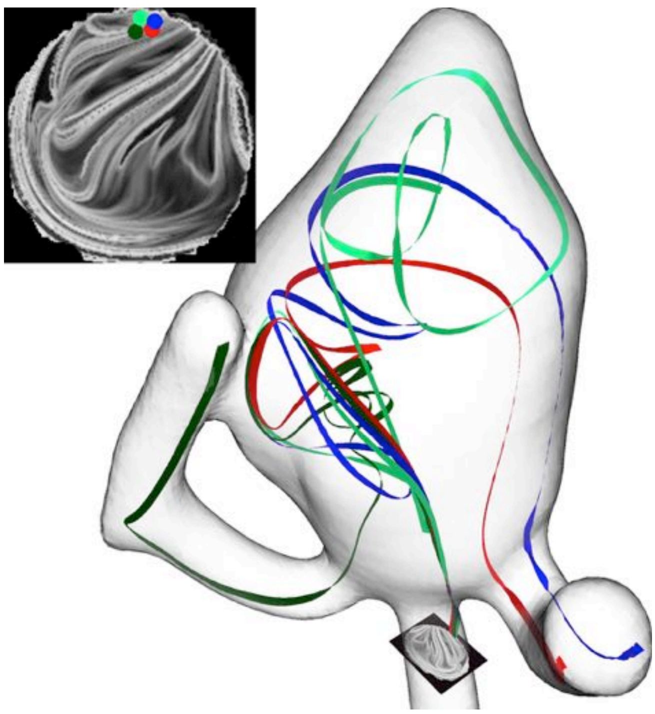




Visualization: Rickard Englund

Coherence maps for flow clustering

Coherence map also shows regions that cannot be meaningfully clustered and gives a general overview over the coherence of the flow.



Visualization: Rickard Englund

Scalar Field Topology in Visualization

Popular concepts

Why do people like scalar field topology so much?

Scalar fields in visualizaiton

Scalar field

- Mapping from domain ***D*** into a set of **scalar attributes** $\mathcal{S} \subset R$.

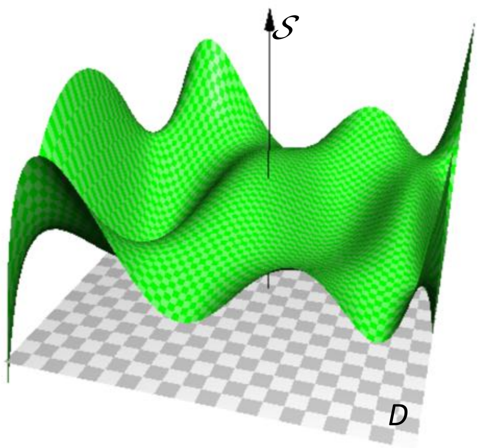
$$s: D \subset \mathbb{R}^n \rightarrow S \subset \mathbb{R}$$

$$D \subset \mathbb{R}: \quad x \mapsto s(x)$$

$$D \subset \mathbb{R}^2: \quad (x,y) \mapsto s(x,y)$$

$$D \subset \mathbb{R}^3: \quad (x,y,z) \mapsto s(x,y,z)$$

2-dimensional scalar field



function plot - height field

Scalar fields in visualizaiton

- Which points *x* in domain *D* have value equal to *S(x) = w* (isovalue)?
  - $D \subset R^2$ → set of points is called isocurve or isocontour *I*
  - $D \subset R^3$ → set of points is called **isosurface** *I*

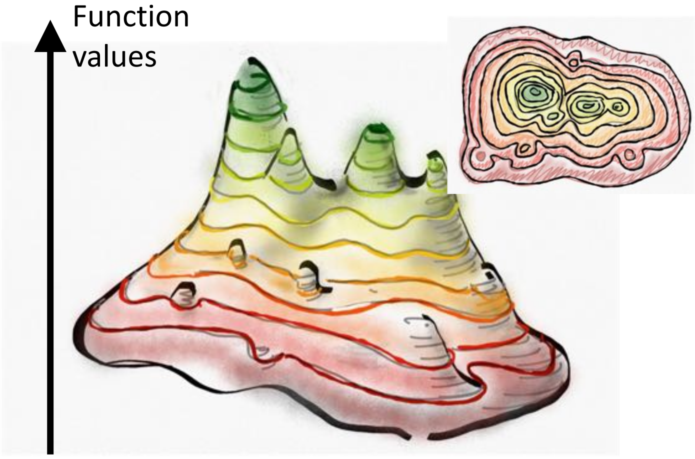
  The isocontour respectively isosurface to *w* is given by $I = S^{-1}(w)$
- What are interesting isovalues?
- Where does the function reach its „maximum values"?
  - Extremal points
  - Ridge and valley lines → **topological analysis**
- Are there any separating surfaces in the data set (scalar value changes rapidly), e.g. material surfaces?
  - Edge detection → **segmentation** methods
  - Automated **transfer function design (color map)** for **volume rendering**
- Are there any specific **patterns, symmetries**?
- …

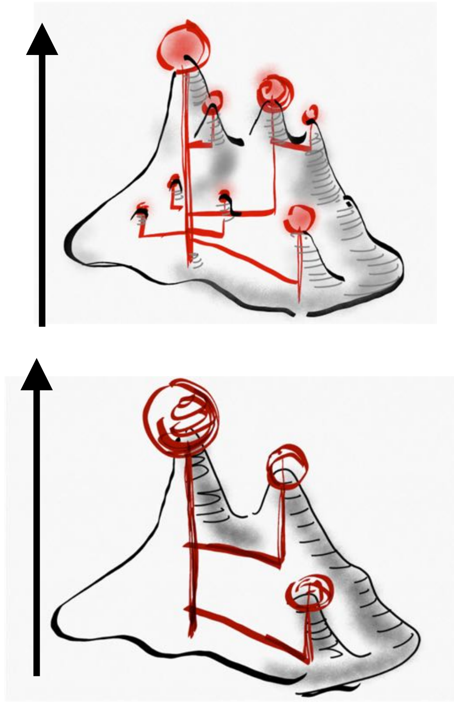
Scalar fields in visualization - Direct Visualization Methods

- **Height fields** (1D and 2D) (function graph)
  Interpret the scalar value as height over the observation space and render the resulting surface.
- **Cutting Planes** (3D) with **Color Mapping**
  - Assign color to every scalar value.
  - Intersect the domain with a plane.
  - Display every point of the plane with the respective color.
- **Direct Volume Rendering** (3D)
  Assign optical properties to every scalar value (emission absorption, etc.) and compute the corresponding image.
- **Isocontour** (2D) resp. **isosurface extraction** (3D)
  Determine and display the curve (surface), representing all points in the plane (space) with corresponding scalar value *w*, i.e., compute $S^{-1}(\{w\})$.

Why do people like scalar field topology so much?

Some concepts are easy to explain - Contour tree





Why do people like scalar field topology so much?

Same concepts are easy to explain - Extremal structures

A sparse subsets of the Morse-Smale complex
Encodes adjacency of extrema

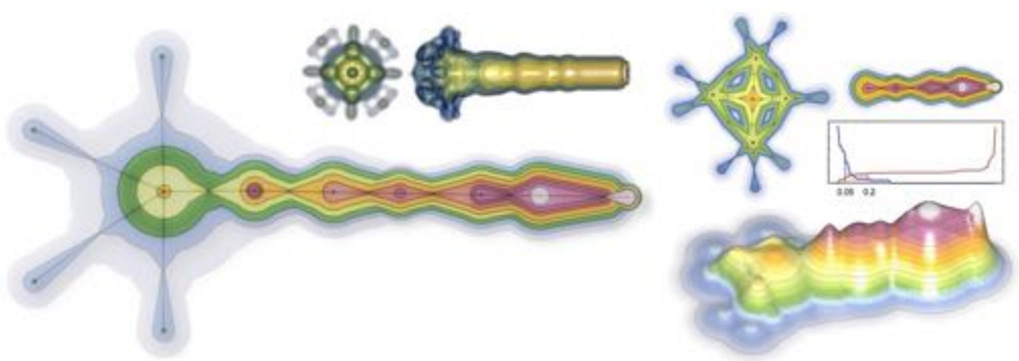
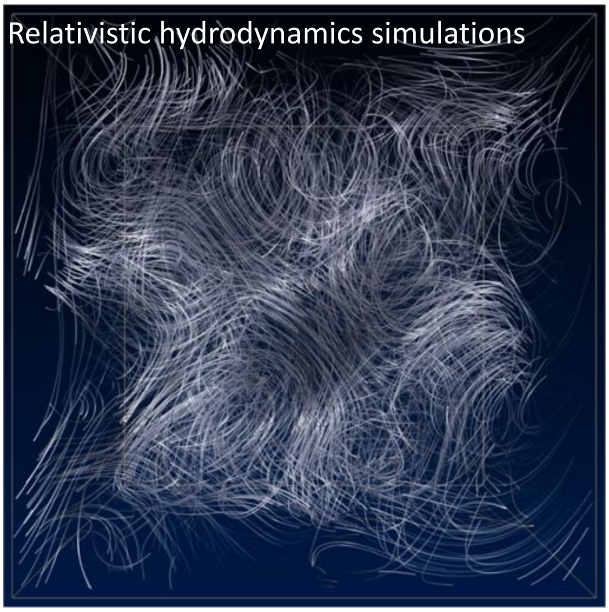
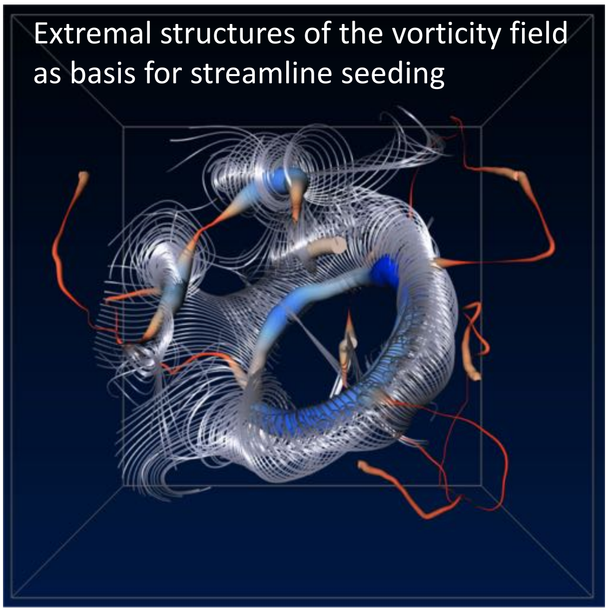


Image: Carlos Correa

Why do people like scalar field topology so much?

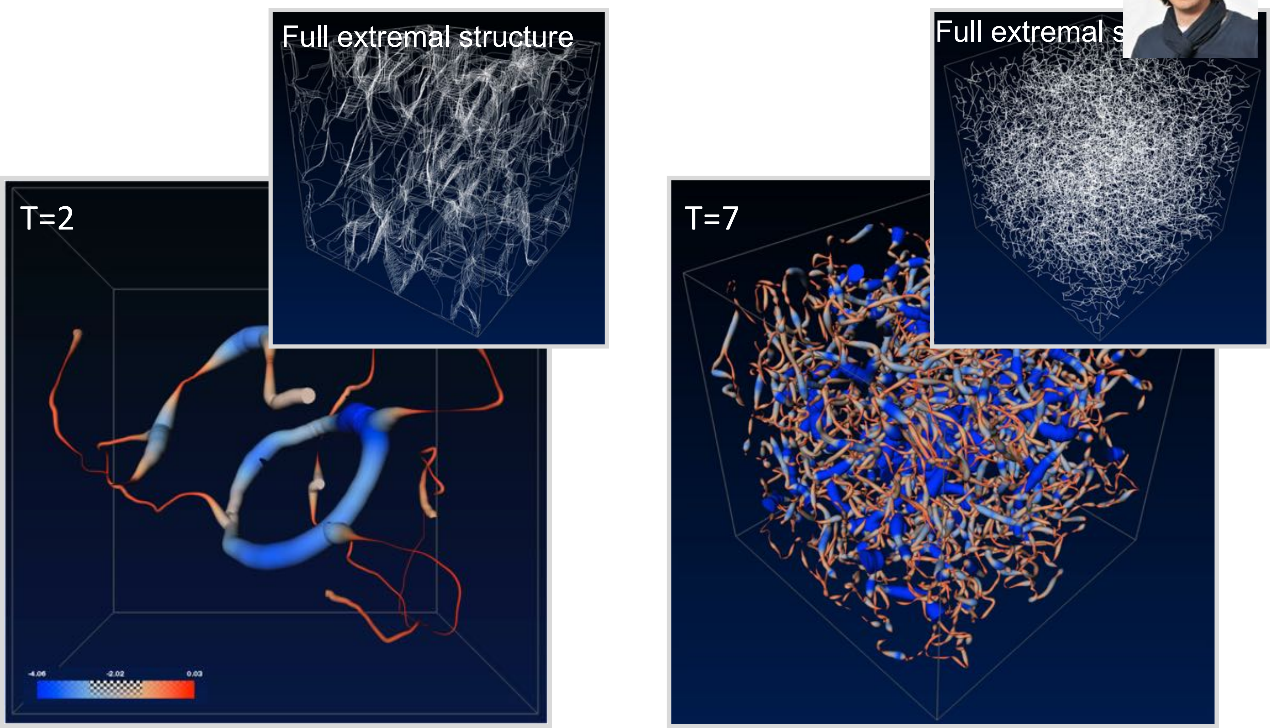
There are more and more convincing examples





Data: Luciano Rezzolla, AEI Potsdam

Why do people like scalar field topology so much



Also the complexity of the topology strongly increases over time

Reininghaus et al. *TADD: A Computational Framework for Data Analysis using Discrete Morse Theory*. 2010

Why do people like scalar field topology so much?

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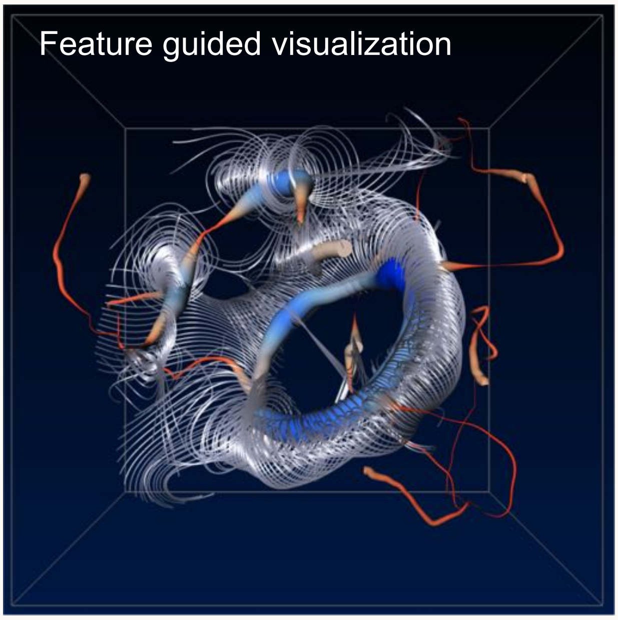

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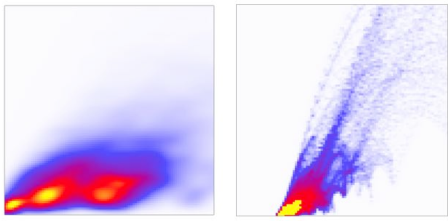
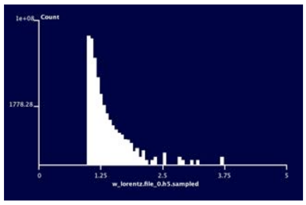
Statistical Analysis + Exploration
E.g. Scatterplots,



Histograms

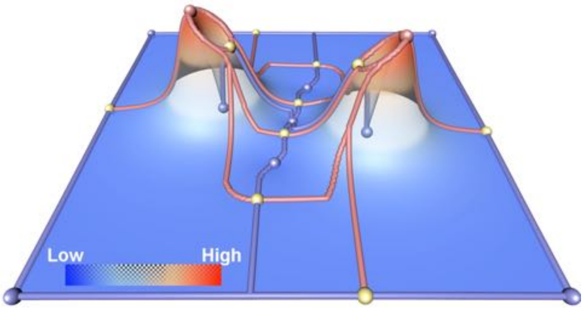


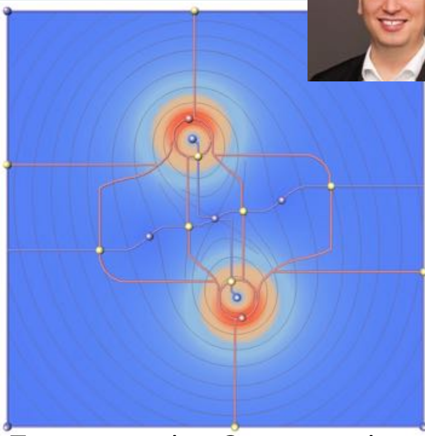
Topology in Applications

Some examples in flow visualization

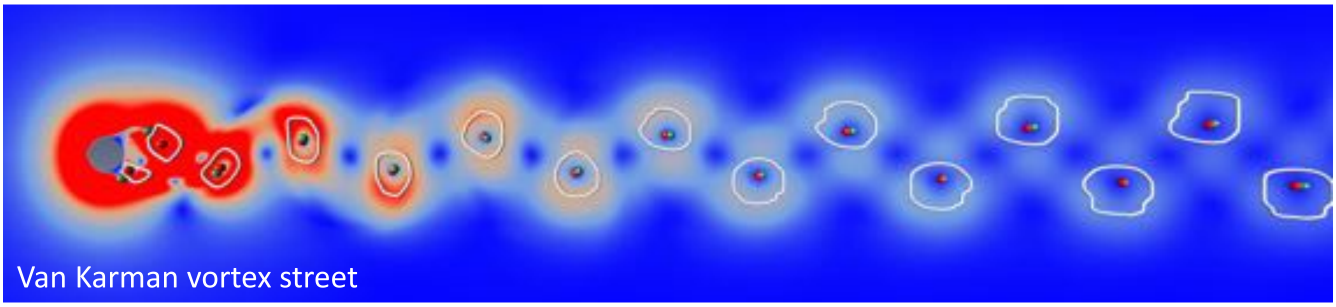
Some basic application in flow visualization

Vortex region extraction based on **extremal structures** of the acceleration magnitude





Two corotating Oseen vortices

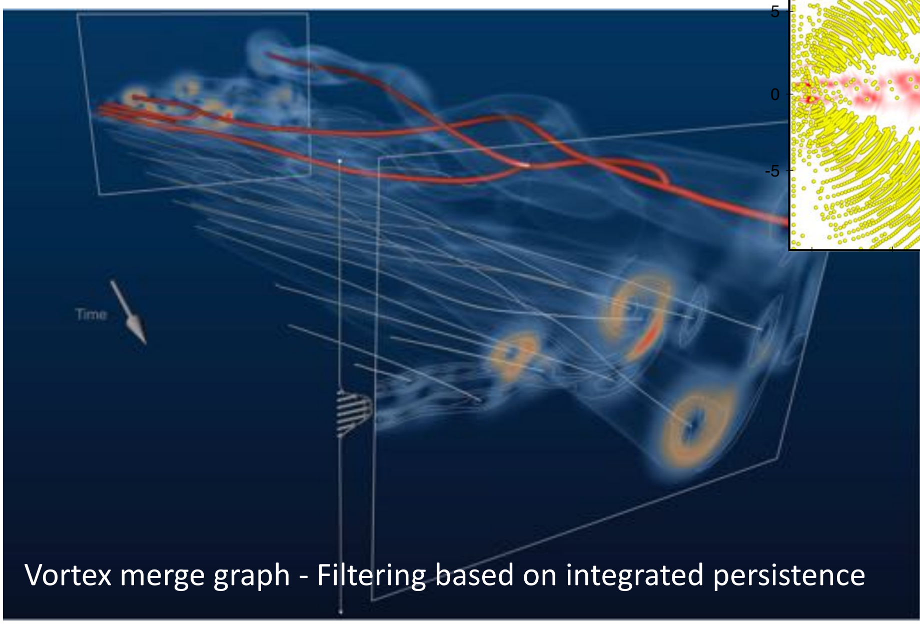


Van Karman vortex street

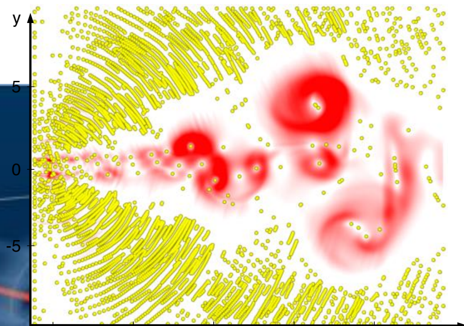
Jens Kasten et al. *Acceleration Feature Points of Unsteady Shear Flows*, 2016
Kasten et al. *Two-dimensional Time-dependent Vortex Regions based on the Acceleration Magnitude*, 2011

Some basic application in flow visualization

Stable tracking of critical points



Vortex merge graph - Filtering based on integrated persistence

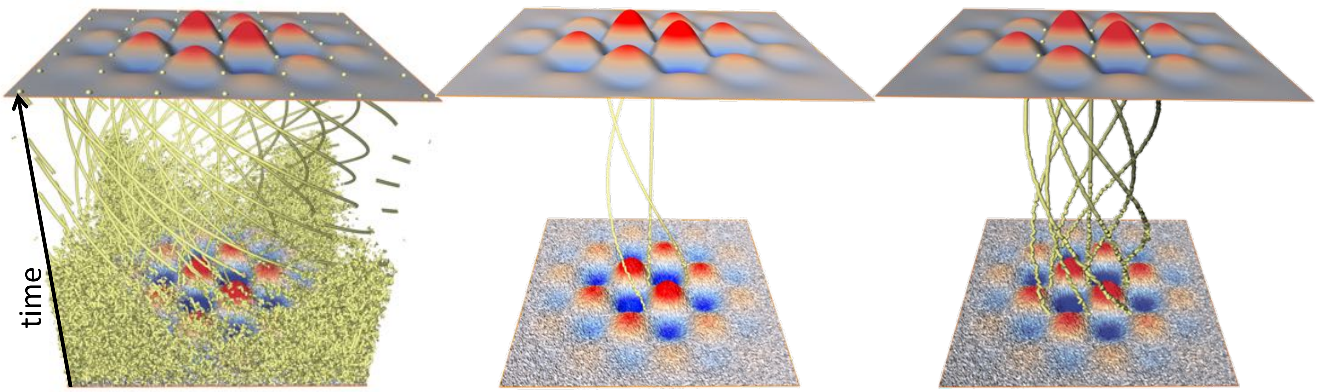


All critical points in one time slice

Kasten et al. Vortex Merge Graphs in Two-dimensional Unsteady Flow Fields. 2012

Stable tracking of critical points

- Analytic function rotated over time
- Amount of noise decreases over time



Numerical tracking
Without filter

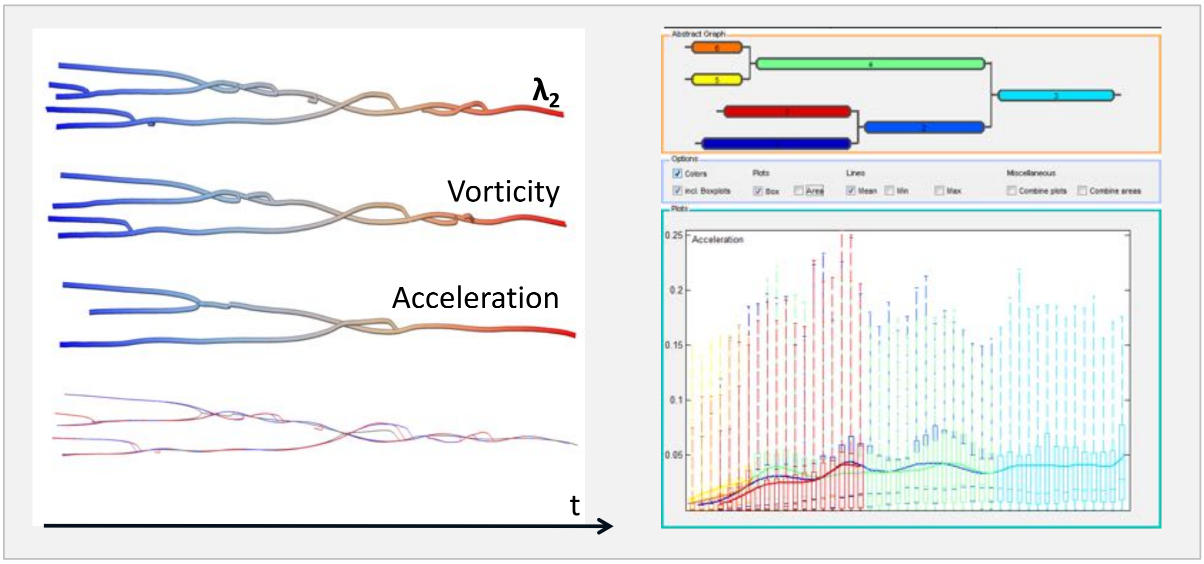
Numerical tracking
Filter: Line length

Combinatorial FFF
Filter: Integrated Persistence

[Jan Reininghaus et al. *Combinatorial feature flow fields: Tracking critical points in discrete scalar fields*, 2011]

Stable tracking of critical points

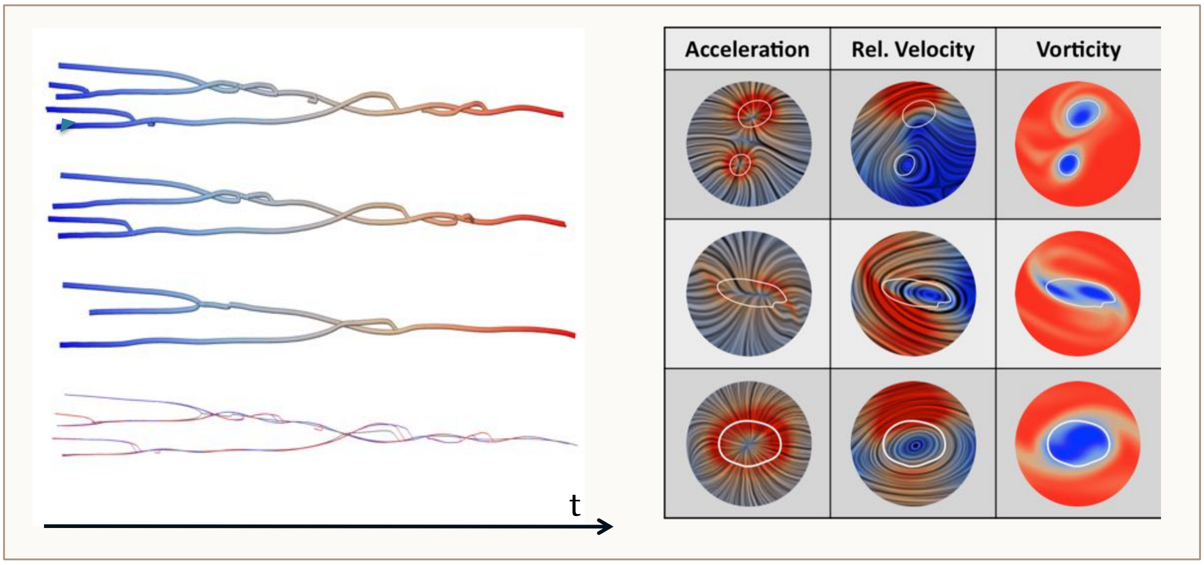
Comparison, abstraction, analysis, quantification, , exploration



Kasten et al. *Analysis of vortex merge graphs*. 2012

Some basic applications

Comparison, abstraction, analysis, quantification, exploration



Visualization: Jens Kasten,

Challenges for the use of topology in Applications

Why do people like scalar field topology so much?

The success of scalar field topology is largely due to

- Explicit feature geometry that can be used for further exploration
- Hierarchical data abstraction (Persistence as importance measure)
- Stable extraction methods
- Rigorous mathematical guaranties
- Comes in many different flavors

However this comes not for free

Algorithmic design decisions

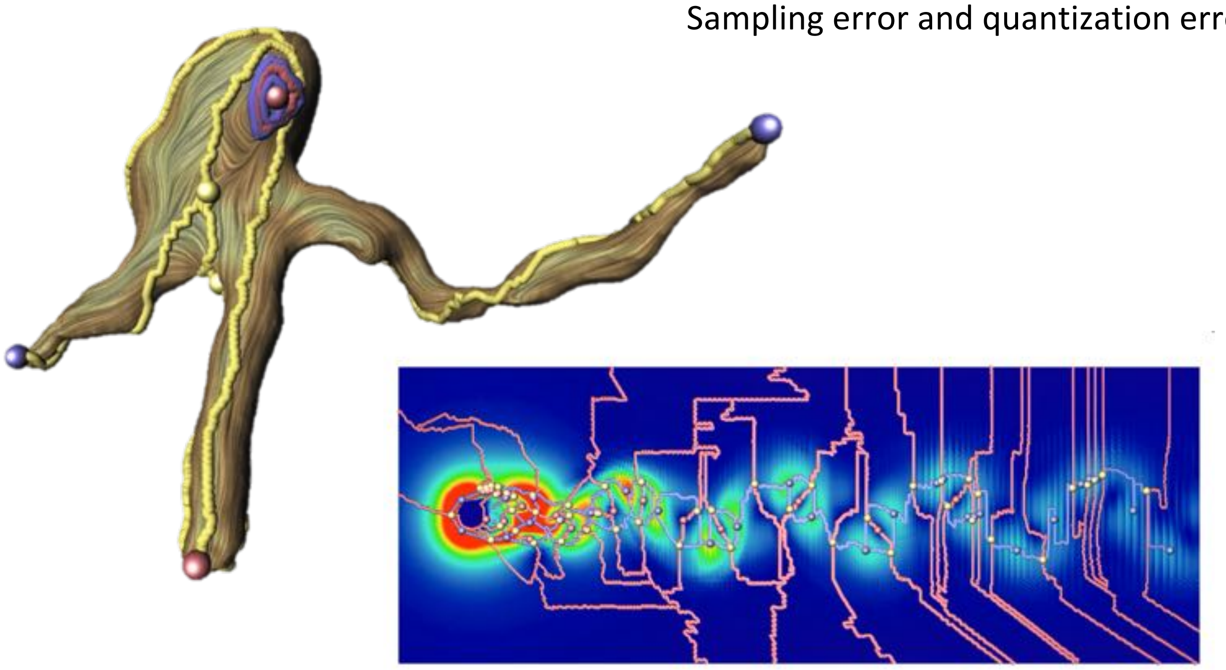
- Simulation of simplicity can introduce artifacts
- Piecewise linear interpolation does not always fit the application needs
- Tracking and simplification does not commute

Conceptual challenges for Topo in Vis

- Geometric embedding is essential for visualization
- **Some people don't want to learn topology, it often must be hidden under familiar concepts**
- What is the right field to explore
- Counting and measuring is not objective

Geometric Embedding of Separatrices

Sampling error and quantization error

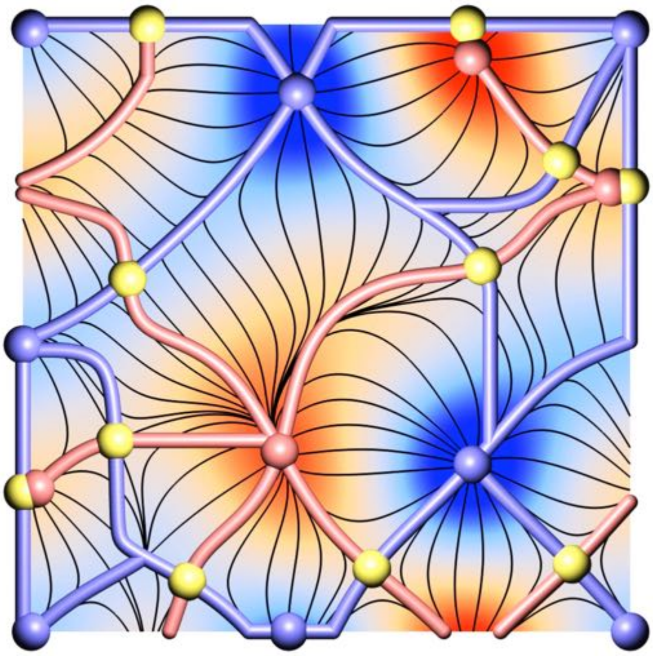


No engineer will accept such images

Reininghaus et al. *Fast Combinatorial Vector Field Topology.* 2011

Geometric Embedding of Separatrices

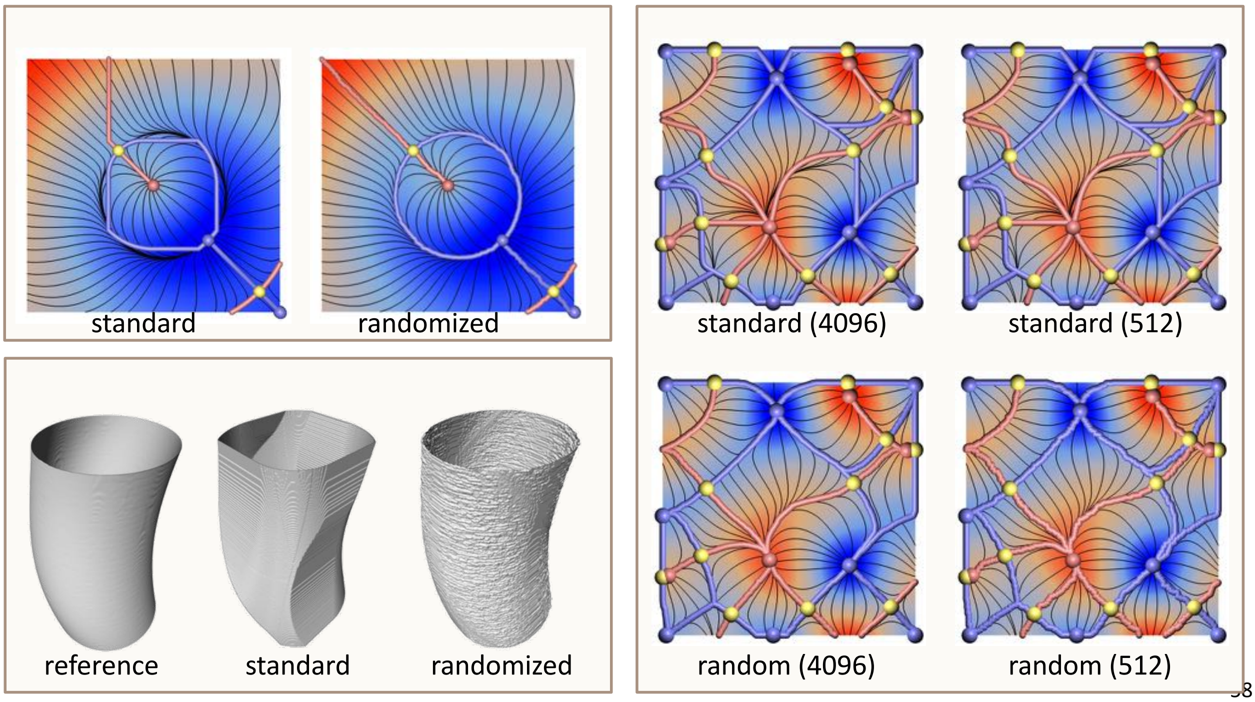
Sampling error and quantization error



Reininghaus et al. *Combinatorial Gradient Fields for 2D Images with Empirically Convergent Separatrices*. 2012

Geometric Embedding of Separatrices – Empirical Convergent Separatrices

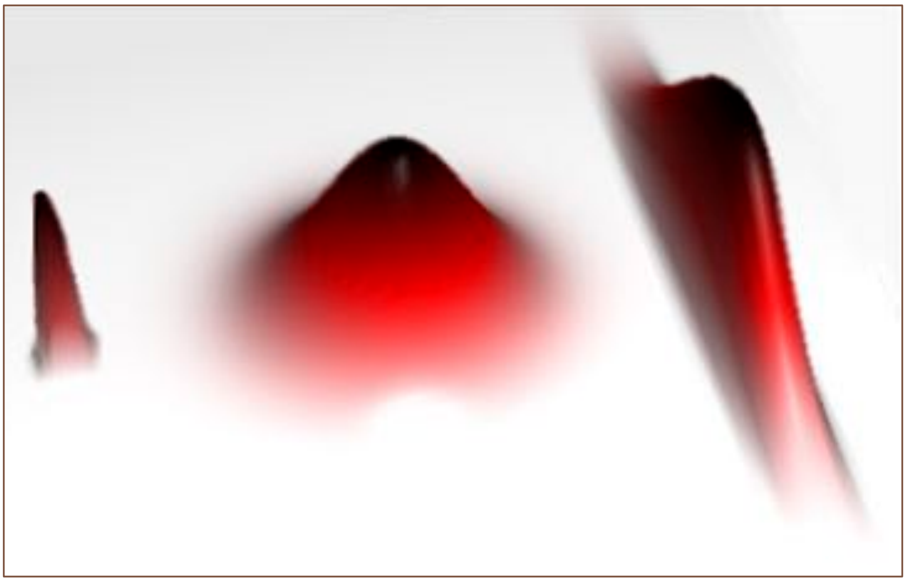
The (continuous) gradient direction cannot be represented exactly,
Pick an edge according to a random variable defined by the data



Visualization: Jan Reininghaus

Geometry and Topology – Scale Space Persistence

Topological stability Persistence
Lifetime of homology classes of sublevel sets



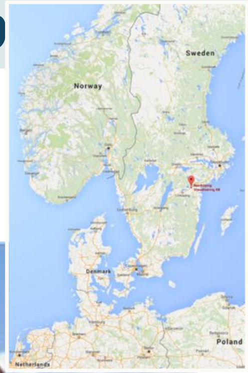
Does not distinguish between different types of maxima
outliers, ridges or hills are treated the same

Reininghaus et al. A Scale Space Based Persistence Measure for Critical Points in 2D Scalar Fields. 2011

Applied topology in visualization

- The goal of visual data analysis and exploration is to generate an environment for scientific reasoning through interaction with data.
- The basis for such an effective environment is a multi-scale data abstraction that can serve as a backbone for data navigation.
- Topological data analysis provides an excellent means for this purpose especially with respect to the rapid development of robust extraction algorithms.
- Mathematical rigorous guarantees contribute strongly to the acceptance of topological analysis tools.
- **However, every application implies new challenges: practical and efficient solutions put into semantic context are needed.**
- Sometimes this might also mean to give up some of the beauty of the mathematical concepts for approximations and heuristics.

TopoInVis workshop in Sweden, June 17-19, 2019



April 15, 2019: deadline for full papers and extended abstracts
May 20, 2019: author notification

Visualization center Norrköping, Linköping University