CS 6170: Computational Topology, Spring 2019
Lecture 27

Topological Data Analysis for Data Scientists

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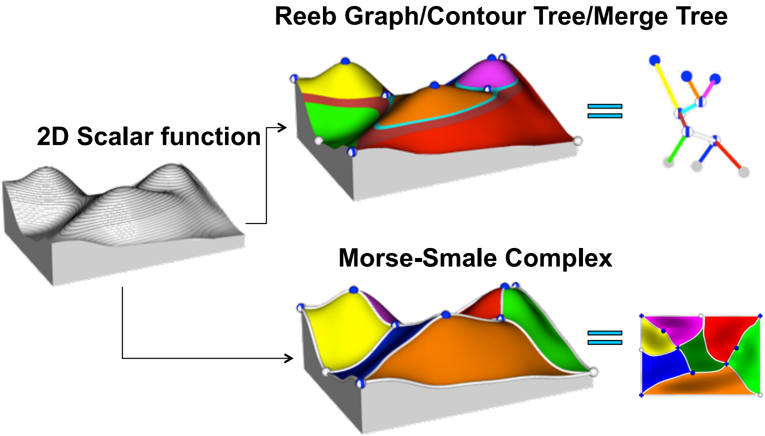
Key development in TDA

1. Abstraction of the data: topological structures
2. Separate features from noise: persistent homology



van Kreveld et al. (1997); Carr et al. (2003); Edelsbrunner et al. (2003a,b)

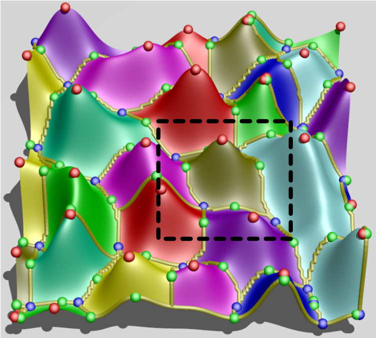
Morse-Smale Complexes

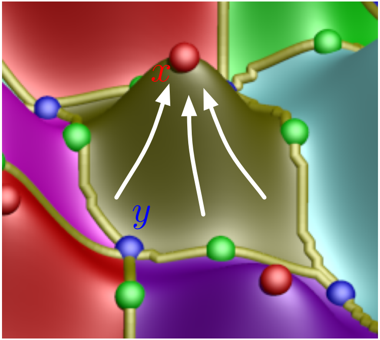
(Edelsbrunner and Harer, 2010, VI.2)

Morse Complex

- $\mathbb{M}$: a smooth manifold embedded in $\mathbb{R}^n$.
- $f : \mathbb{M} \to \mathbb{R}$: a smooth function with gradient ∇f.
- A point $x \in \mathbb{M}$ is called *critical* if $\nabla f(x) = 0$; otherwise it is *regular*.
- At any regular point x, the gradient is well defined and integrating it in both ascending and descending directions traces out an *integral line*, which is a maximal path whose tangent vectors agree with the gradient.
- Each integral line begins and ends at critical points.
- The *ascending manifolds* of a critical point p are defined as all the points whose integral lines **start** at p.
- The *descending manifolds* of a critical point p are defined as all the points whose integral lines **end** at p.
- The ascending (descending) manifolds decompose the domain into cells.
- These cells form a complex called a *Morse complex* of f $(-f)$.

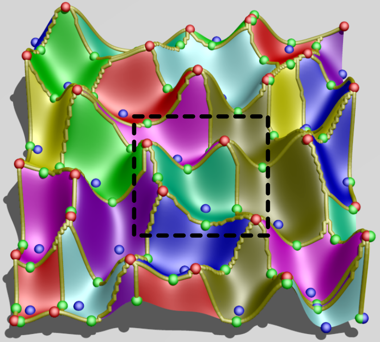
All the points whose integral lines **end** at a critical point x.

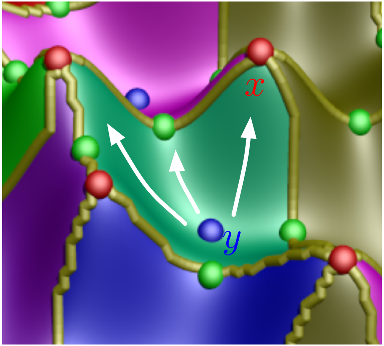




Ascending Manifolds

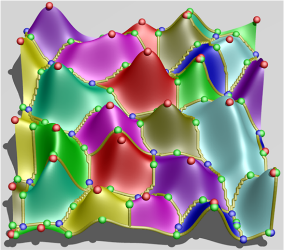
All the points whose integral lines **start** at a critical point y.



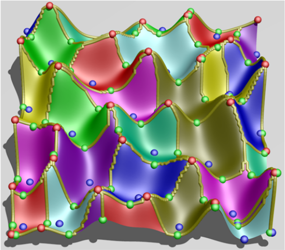


Morse-Smale Complex

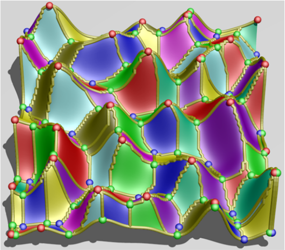
- The set of intersections of ascending and descending manifolds creates the *Morse-Smale complex* of f.
- A partition of the data into monotonic regions.



Descending Manifolds
(Unstable Manifolds)



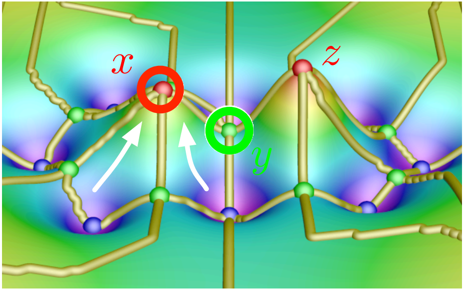
Ascending Manifolds
(Stable Manifolds)

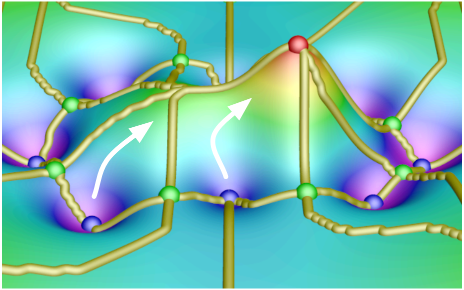


Morse-Smale Complexes

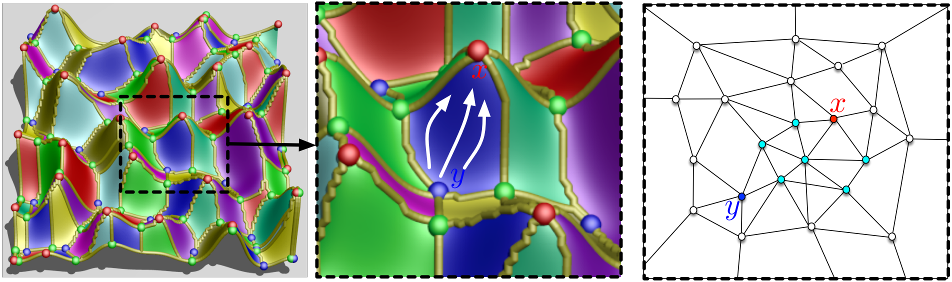
Edelsbrunner et al. (2003a,b)

Persistence Simplification of Morse-Smale Complex





Morse-Smale Complex: approximation in HD



(a) (b) (c)

Applications of Morse-Smale Complexes

Terrain simplification

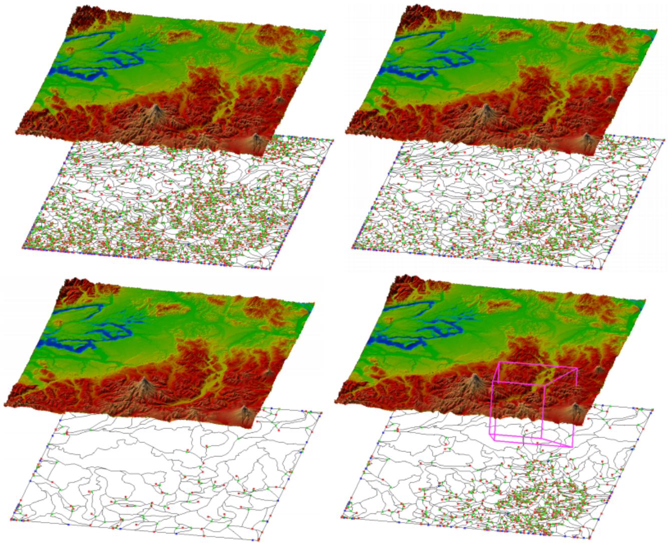


Figure 11: (Upper-left) Puget Sound data after topological noise removal. (Upper-right) Data at persistence of 1.2% of the maximum height. (Lower-left) Data at persistence 20% of the maximum height. (Lower-right) View-dependent re□nement (purple: view frustum).

Bremer et al. (2003)

Morse-Smale Regression

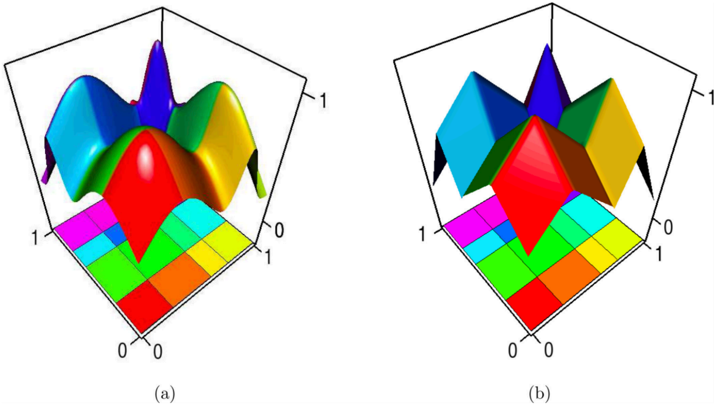


Figure 1.
(a) Illustration of the Morse-Smale complex decomposition of a 2D function and (b) piecewise linear model fit.

Gerber et al. (2012)

Morse-Smale Regression

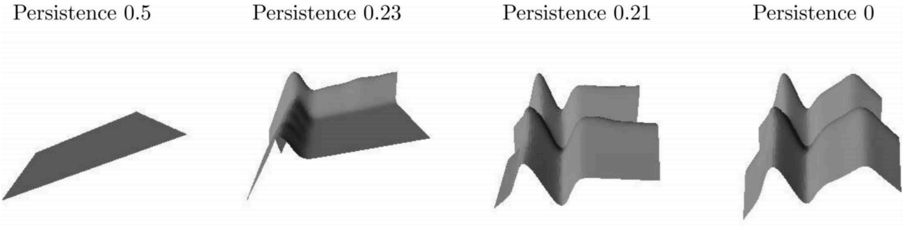
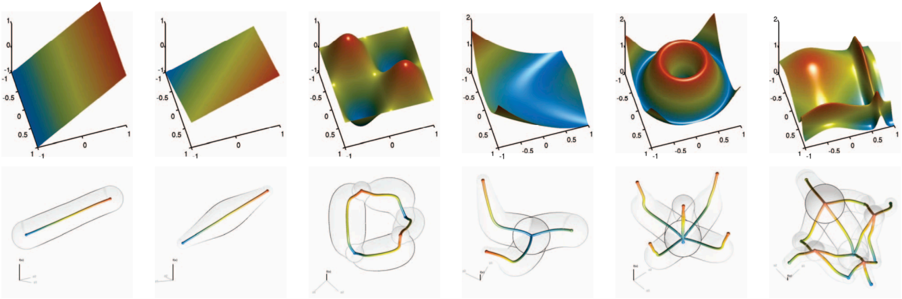


Figure 3.
A hierarchy of regression models induced by the persistence simplification of the Morse-Smale complex. Starting at the highest persistence, with a single minimum and maximum, on the left, to multiple extrema, at zero persistence, on the right.

Gerber et al. (2012)

Visual exploration of HD scalar functions



Gerber et al. (2010)

Nuclear Engineering: Sensitivity Analysis

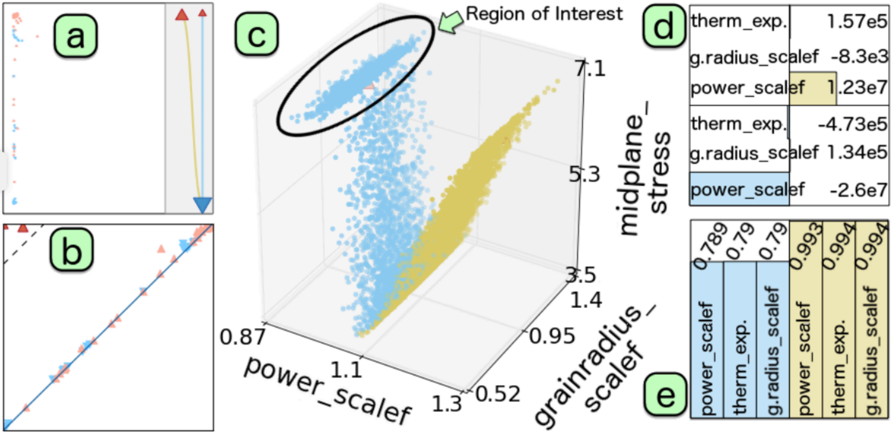
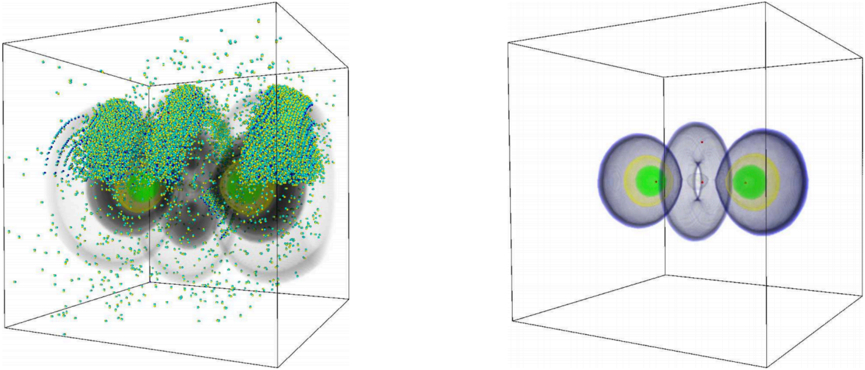


Figure 5: SA of the new nuclear fuel dataset: (a) topology map, (b) persistence diagram, (c) linked scatter plot projection, (d) linear coefficients, and (e) fitness view with stepwise R^2 scores.

Maljovec et al. (2016)

Topological simplification: hydrogen data set



Gyulassy (2007)

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