Advanced Data Visualization

CS 6965

Spring 2018

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University of Utah

Lecture 26

https://www.nytimes.com/2018/04/11/technology/personaltech/i-downloaded-the-information-that-facebook-has-on-me-yikes.html

Edge Bundling Graph-theoretic Measures

NV

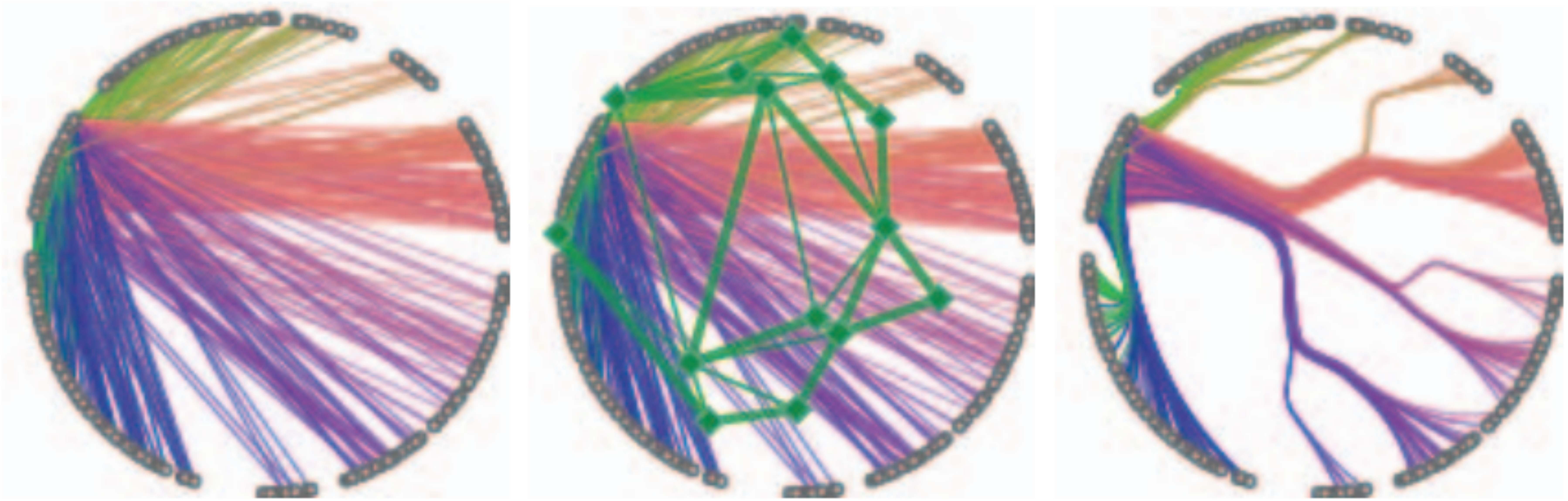
Edge Bundling

Survey: ZhouXuYuan2013

Survey: http://www.chaofz.me/asset/file/Edge%20Bundling%20Survey.pdf [Zhou2017]

Geometry-Based Edge Bundling

Geometry Based Edge Clustering



(a) (b) (c)

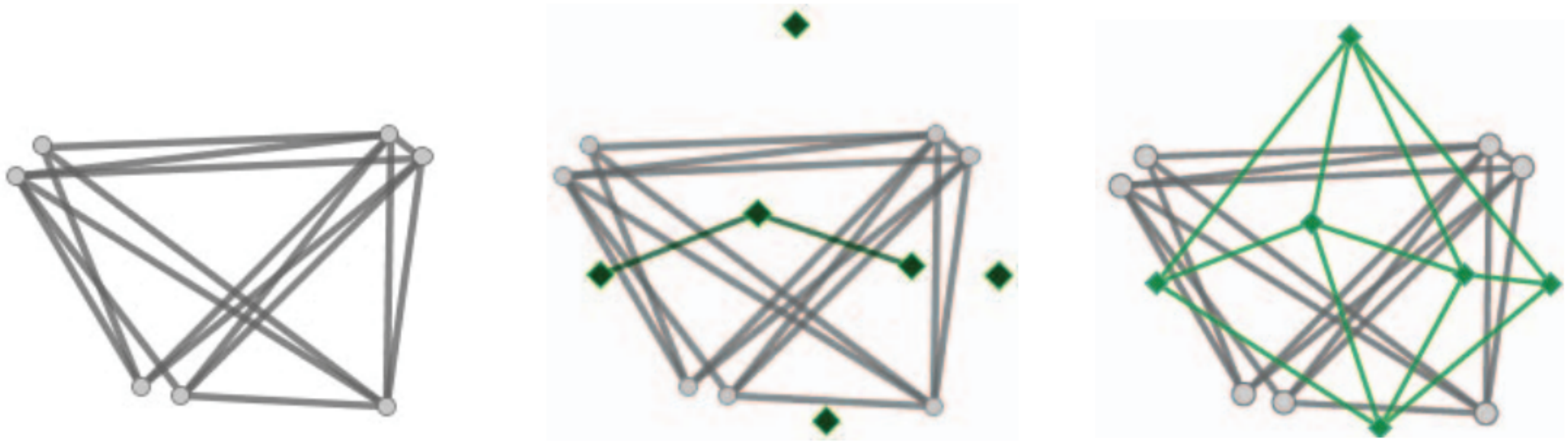
CuiZhouQu2008

Geometry Based Edge Clustering

CuiZhouQu2008

- Graph analyzer: uses edge distribution patterns to figure out representative primary edge directions.
- Mesh generator: generates control-mesh edges perpendicular to each selected primary direction.
- Bundler: uses intersections between the original graph and the control mesh, it sets some control points on the control-mesh edges and curves the original graph edges to pass through these control points to form edge clusters.
- Edge smoother: curved edges with too many zigzags are further fine-tuned to become visually pleasing.

Manual/automatic mesh generation



(a) (b) (c)

Fig. 3. Manual mesh generation: (a) a graph; (b) users click a set of vertices and edges; (c) a mesh is generated by Constrained Delaunay triangulation of the vertices and edges.

CuiZhouQu2008

Animation

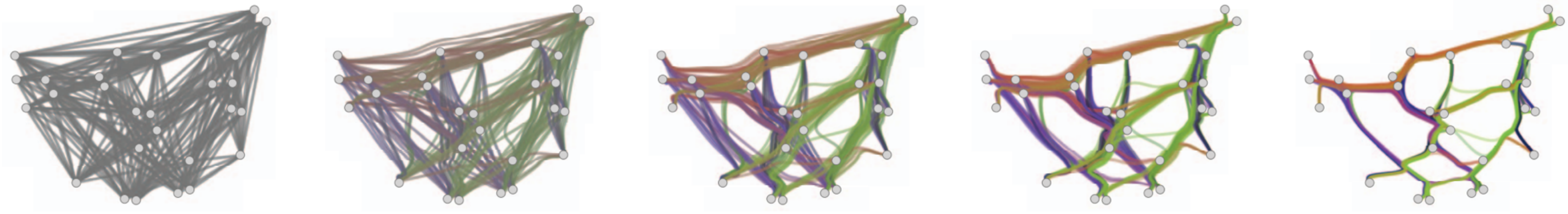


Fig. 10. An animation sequence for an edge-clustering process. The color is used to encode the edge directions.

By viewing the animations, users will have
a better idea about the data and may detect some patterns that may
otherwise disappear in the final static layouts.

CuiZhouQu2008

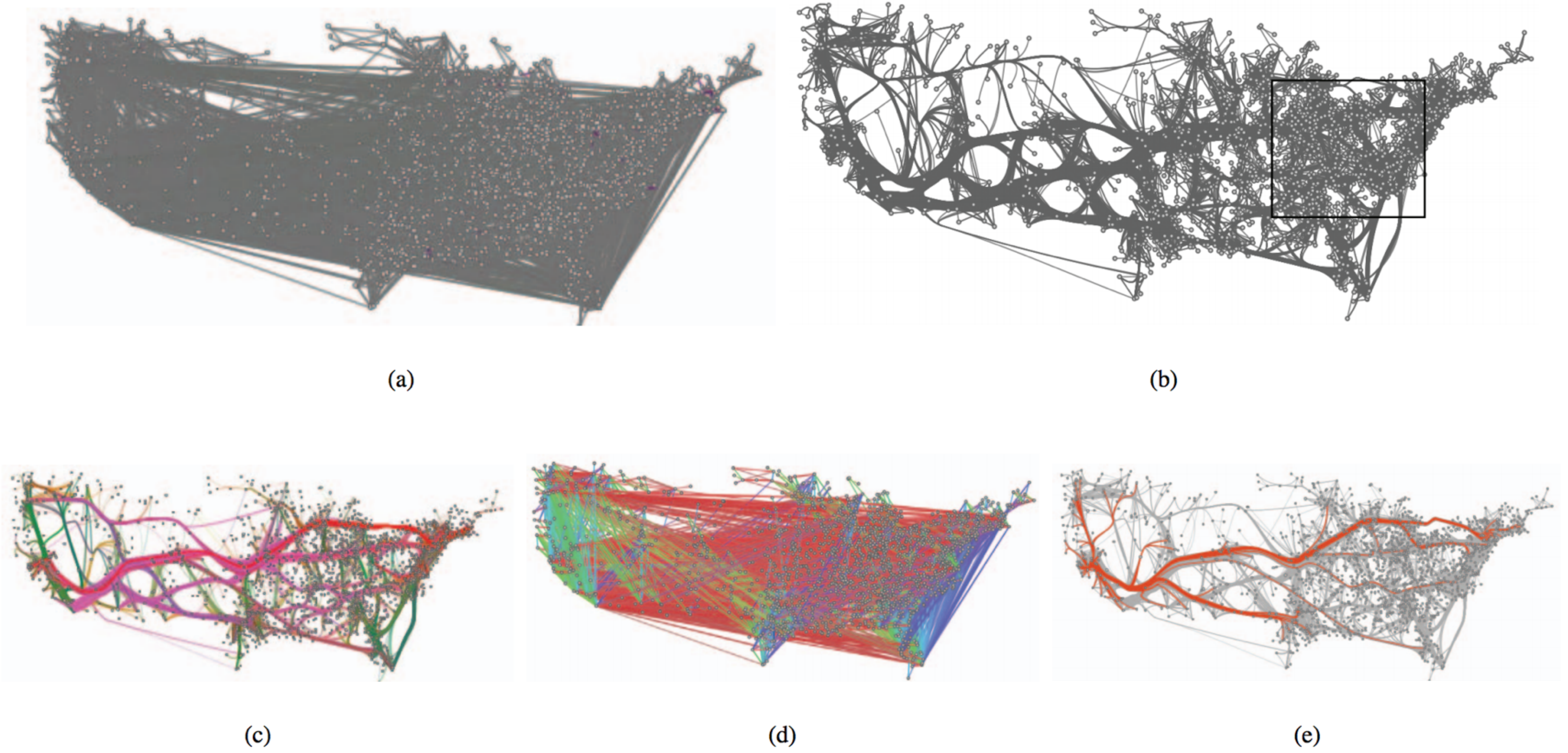


Fig. 14. U.S. immigration graph with 1790 nodes and 9798 edges: (a) original layout; (b) the edge-clustered result; (c) the result after applying edge clustering and transfer function; (d) the result after applying only transfer function; (e) a flow map layout highlighted in orange color.

Force-Directed Edge Bundling

Force-Directed Edge Bundling

- Highlight: A self-organizing approach: edges are modeled as flexible springs having attracting forces on other nodes while the node positions still keep fixed

HoltenWijk2009

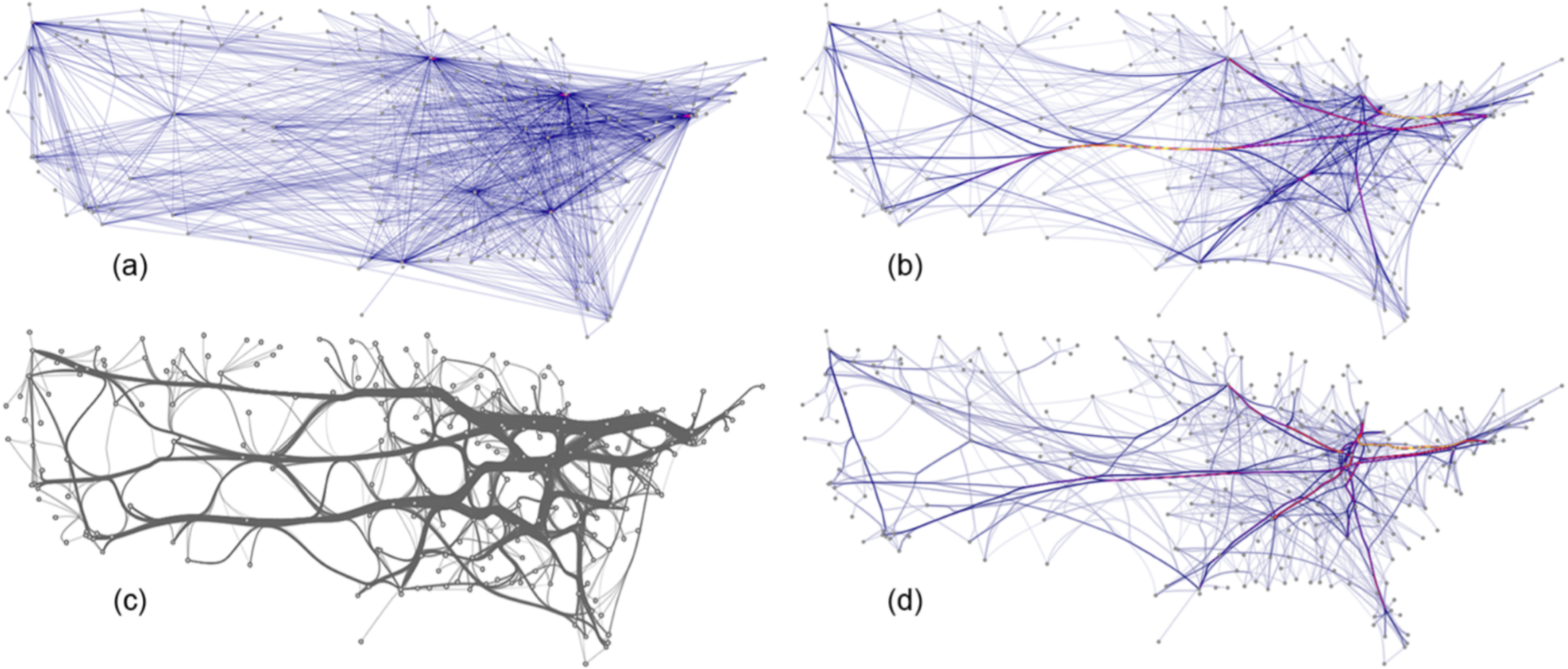


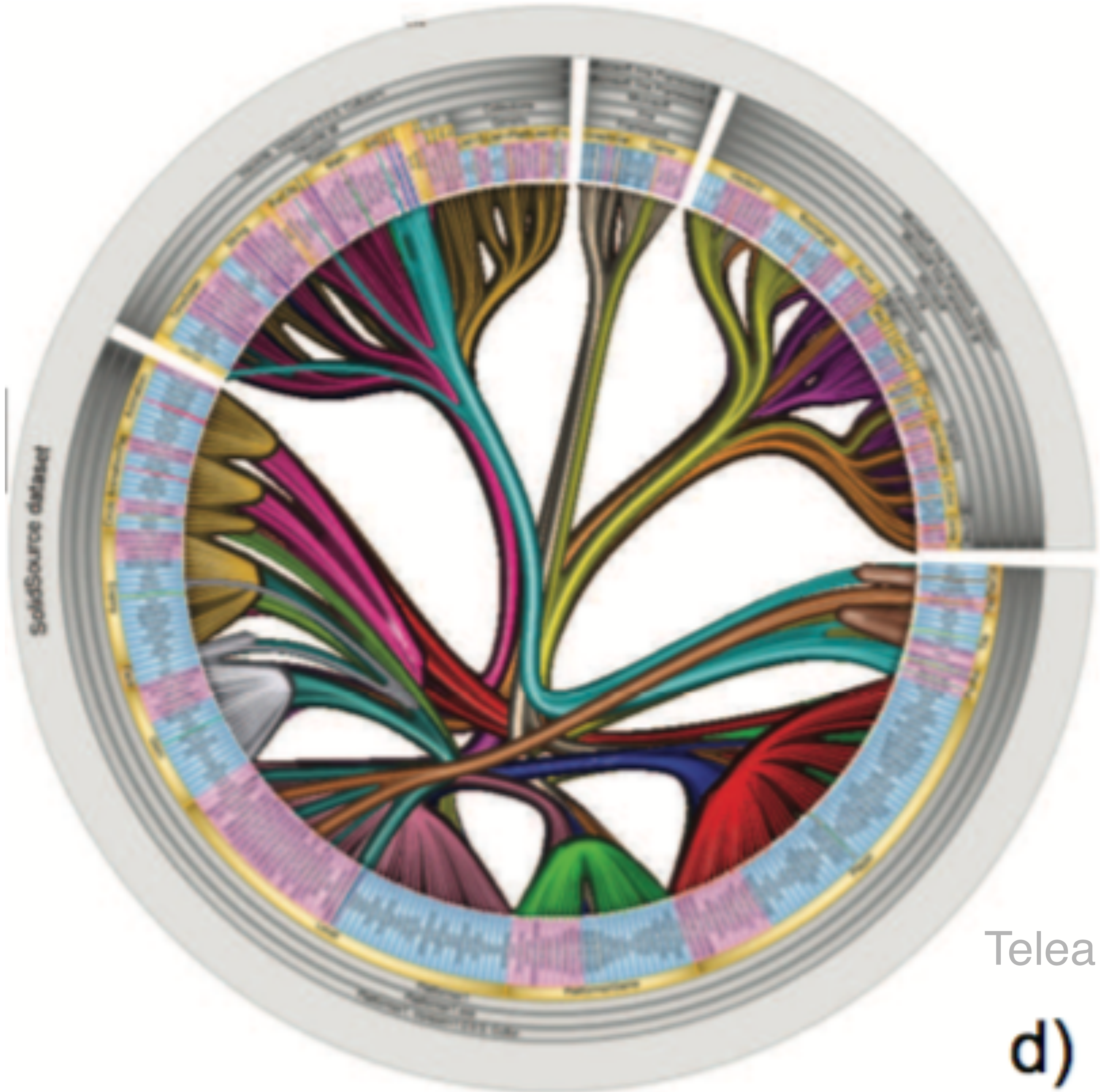
Figure 7: *US airlines graph (235 nodes, 2101 edges) (a) not bundled and bundled using (b) FDEB with inverse-linear model, (c) GBEB, and (d) FDEB with inverse-quadratic model.*

GBEB: Geometry based edge bundling

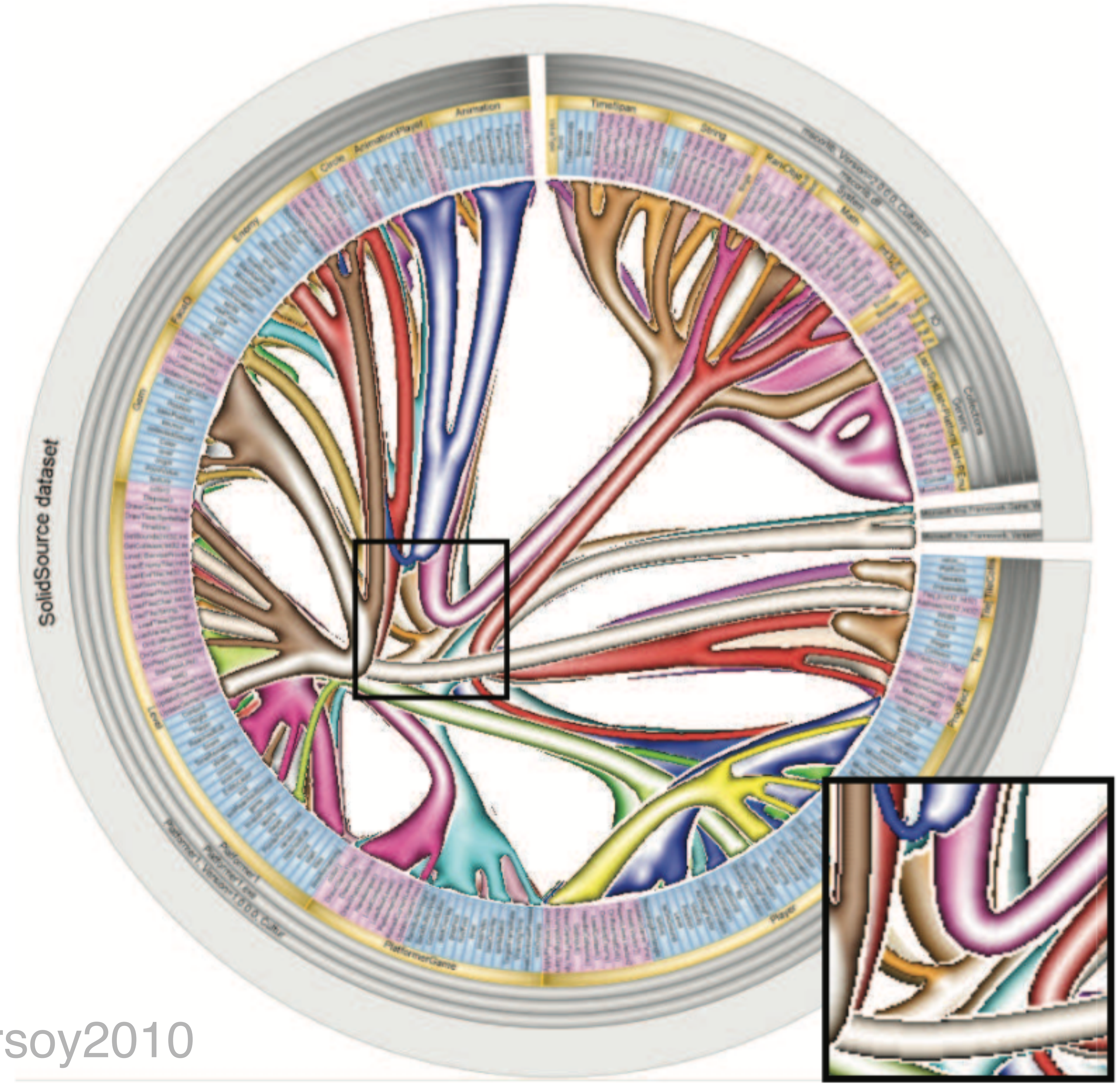
HoltenWijk2009

Image-Based Edge Bundling

Image Based Edge Clustering



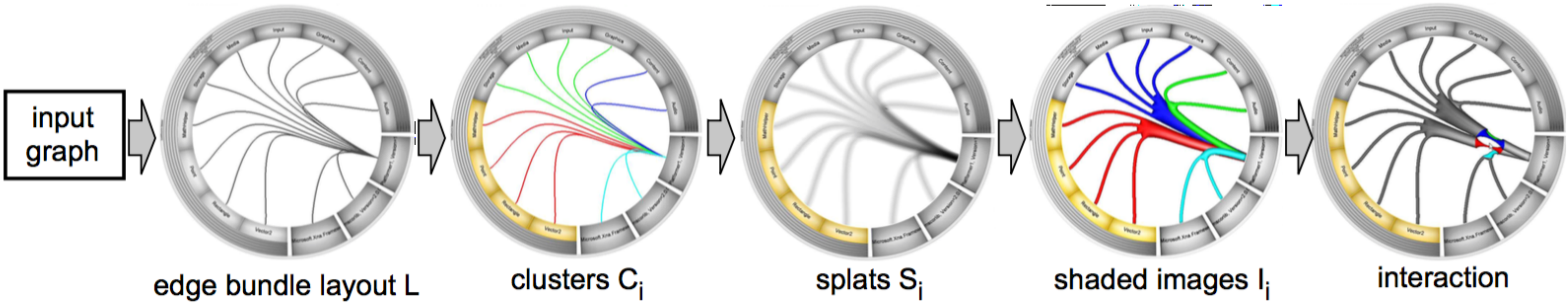
d)



TeleaErsoy2010

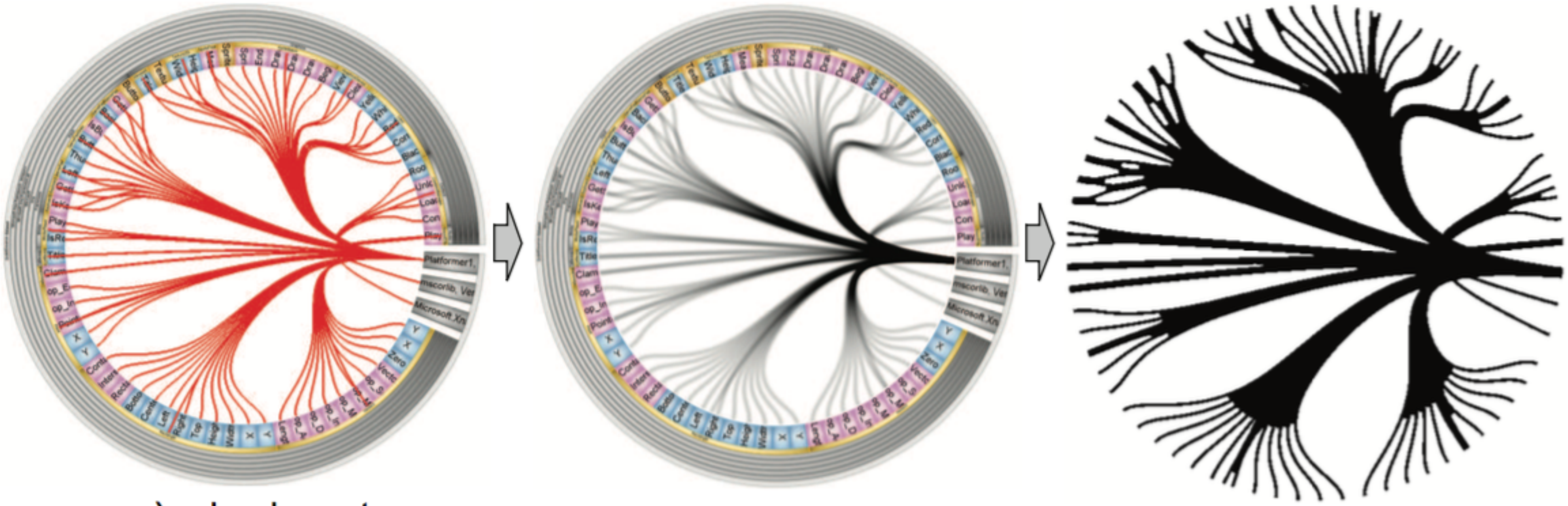
Figure 6: *Bundle visual separation using halos*

Image Based Edge Clustering



TeleaErsoy2010

Shape Construction



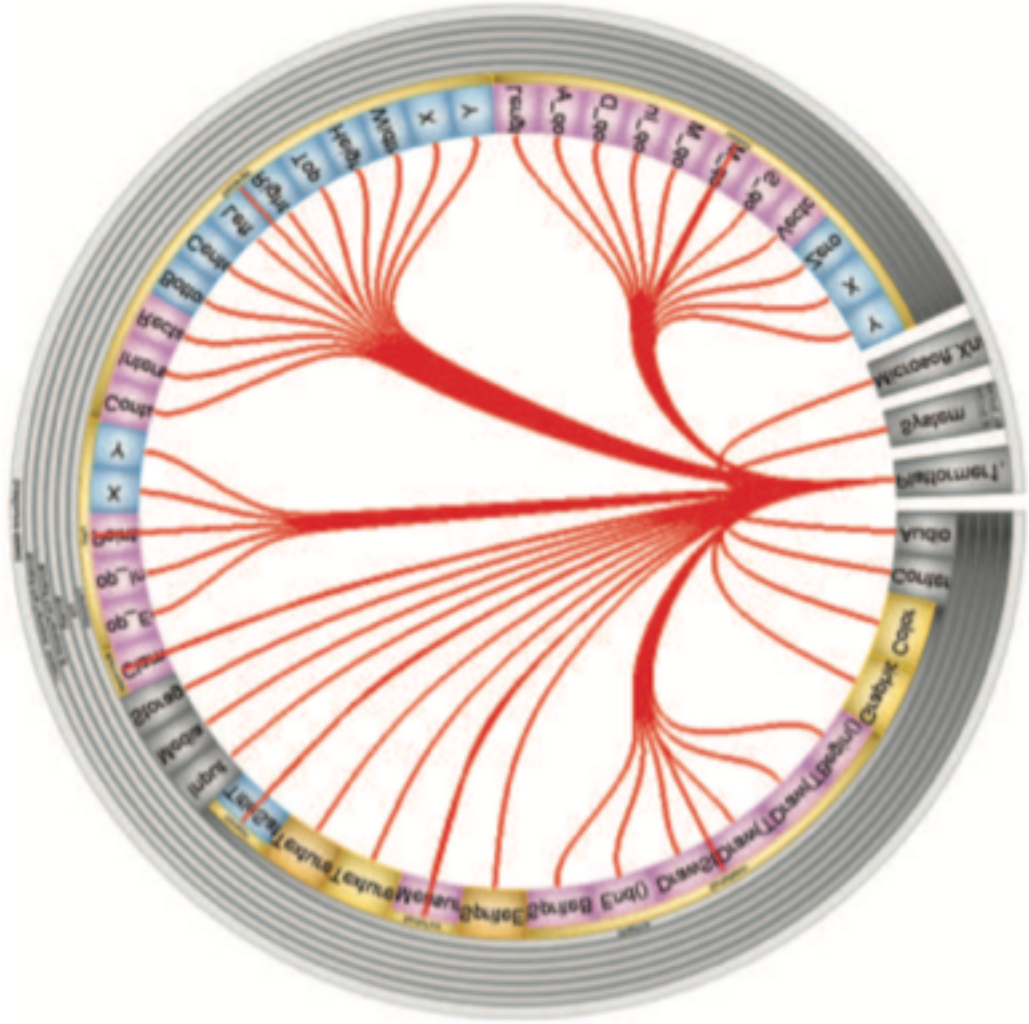
a) edge layout

b) splatted image

c) binary shape

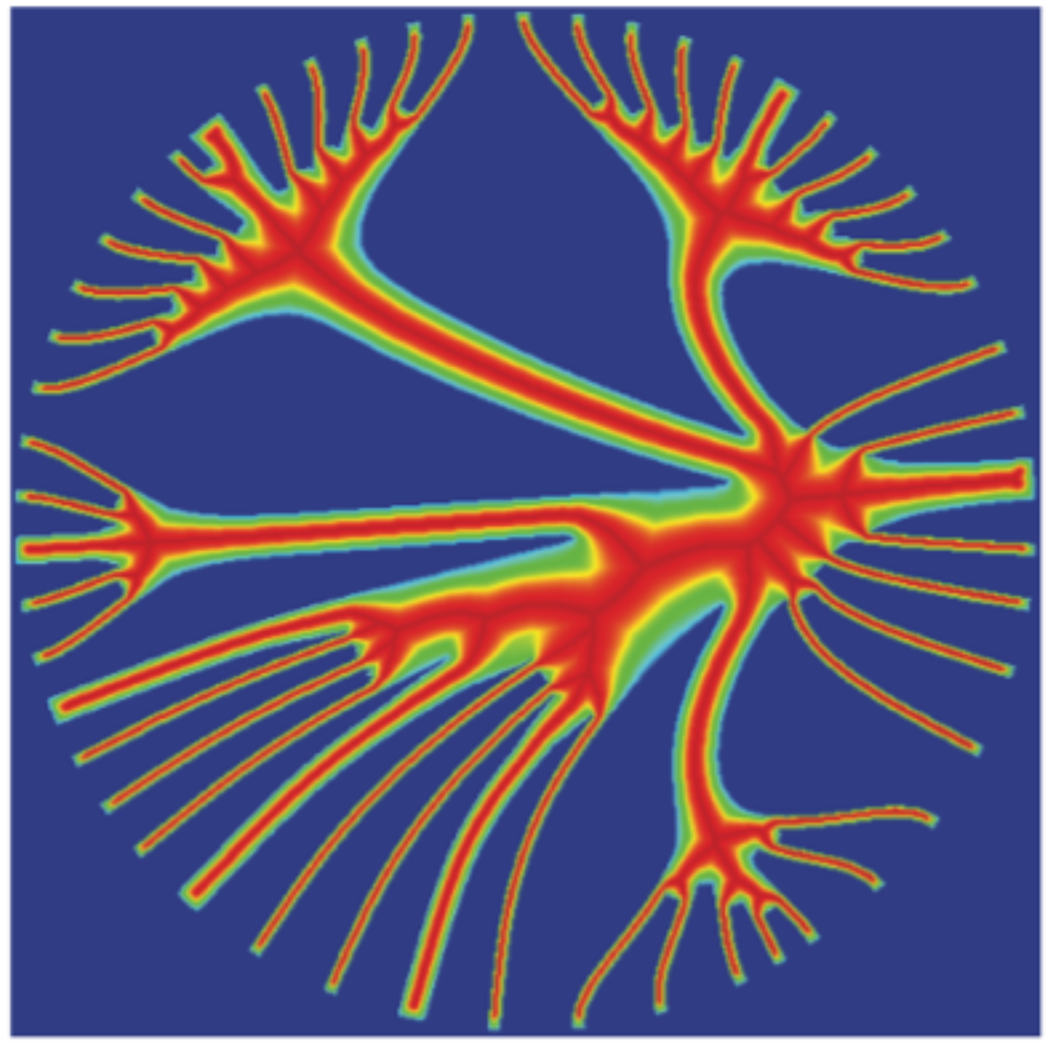
TeleaErsoy2010

Shading

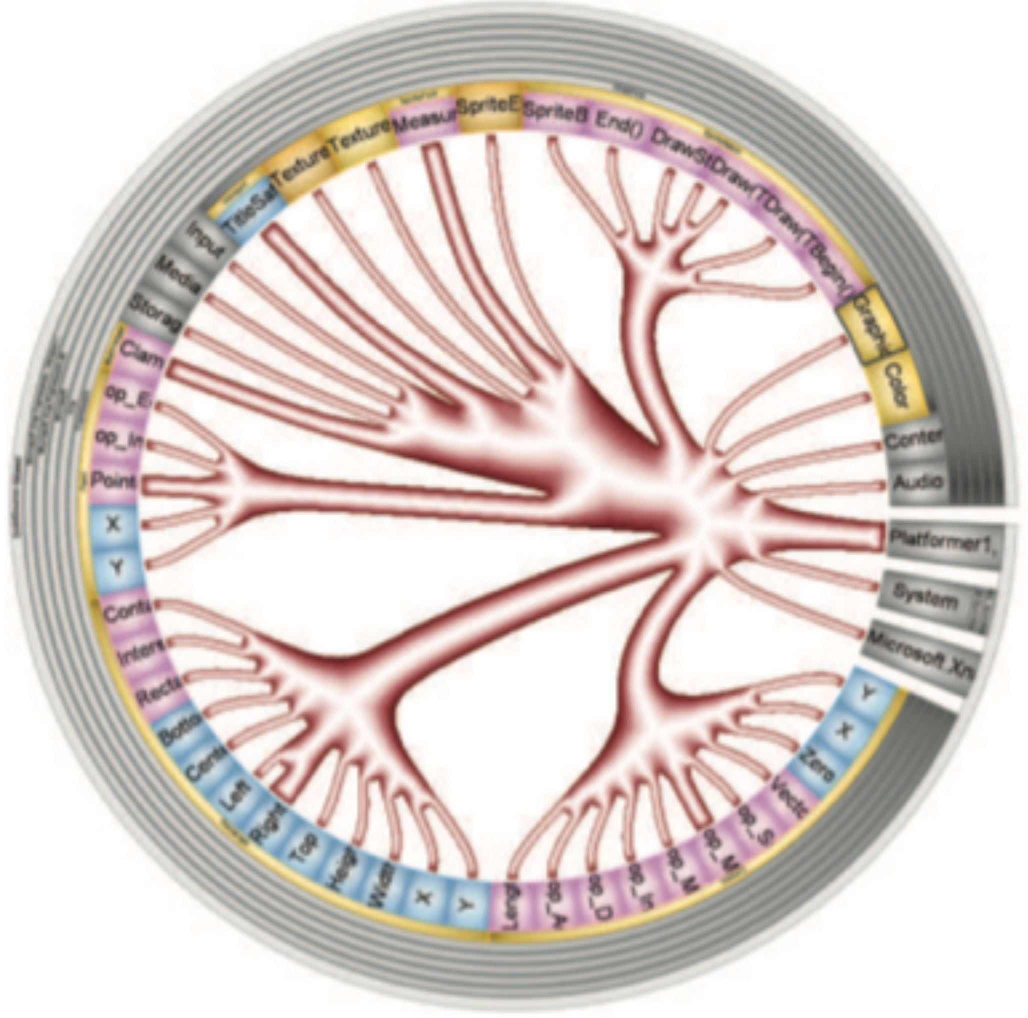


a) edge layout

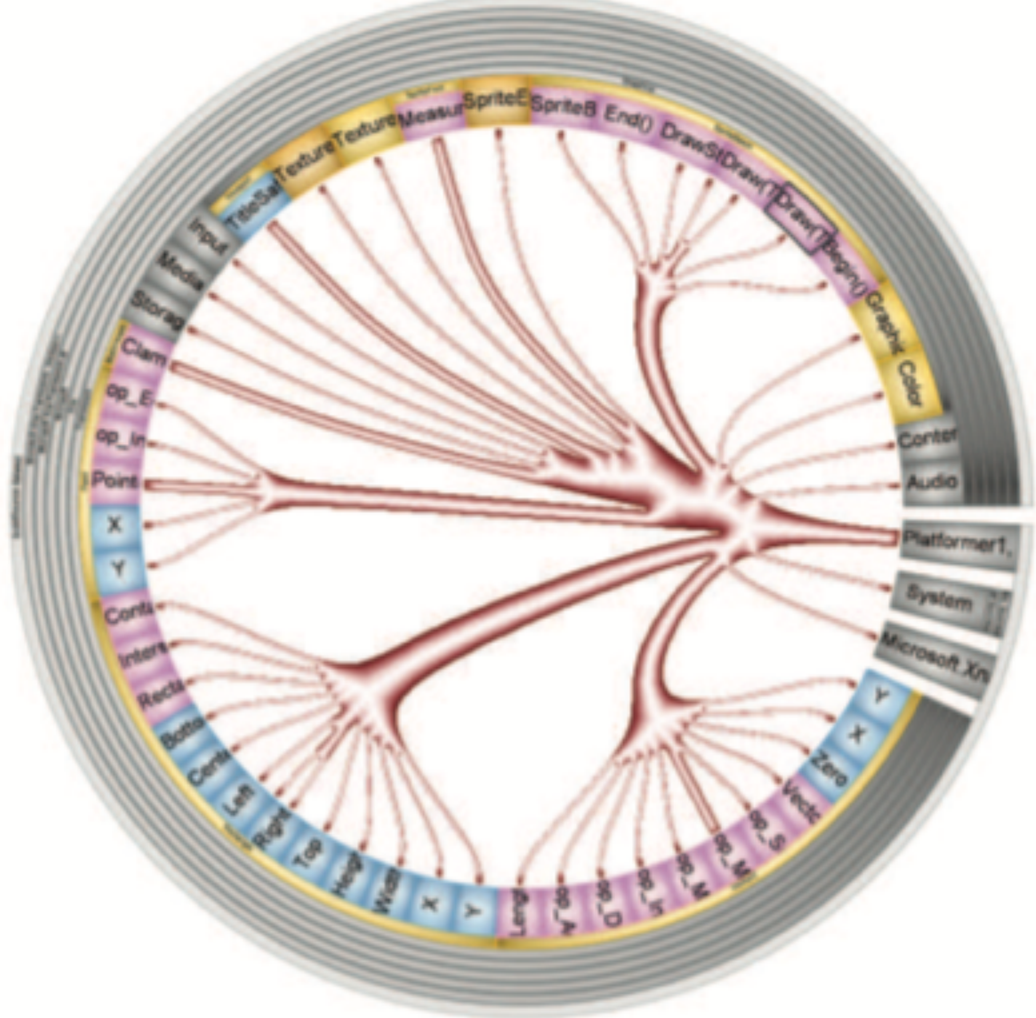
b) shape I and skeleton Sk



c) height profile H



d) convex shading
(large splat size)



e) convex shading
(small splat size)

f) convex shading
(thin shapes)

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Different rendering styles

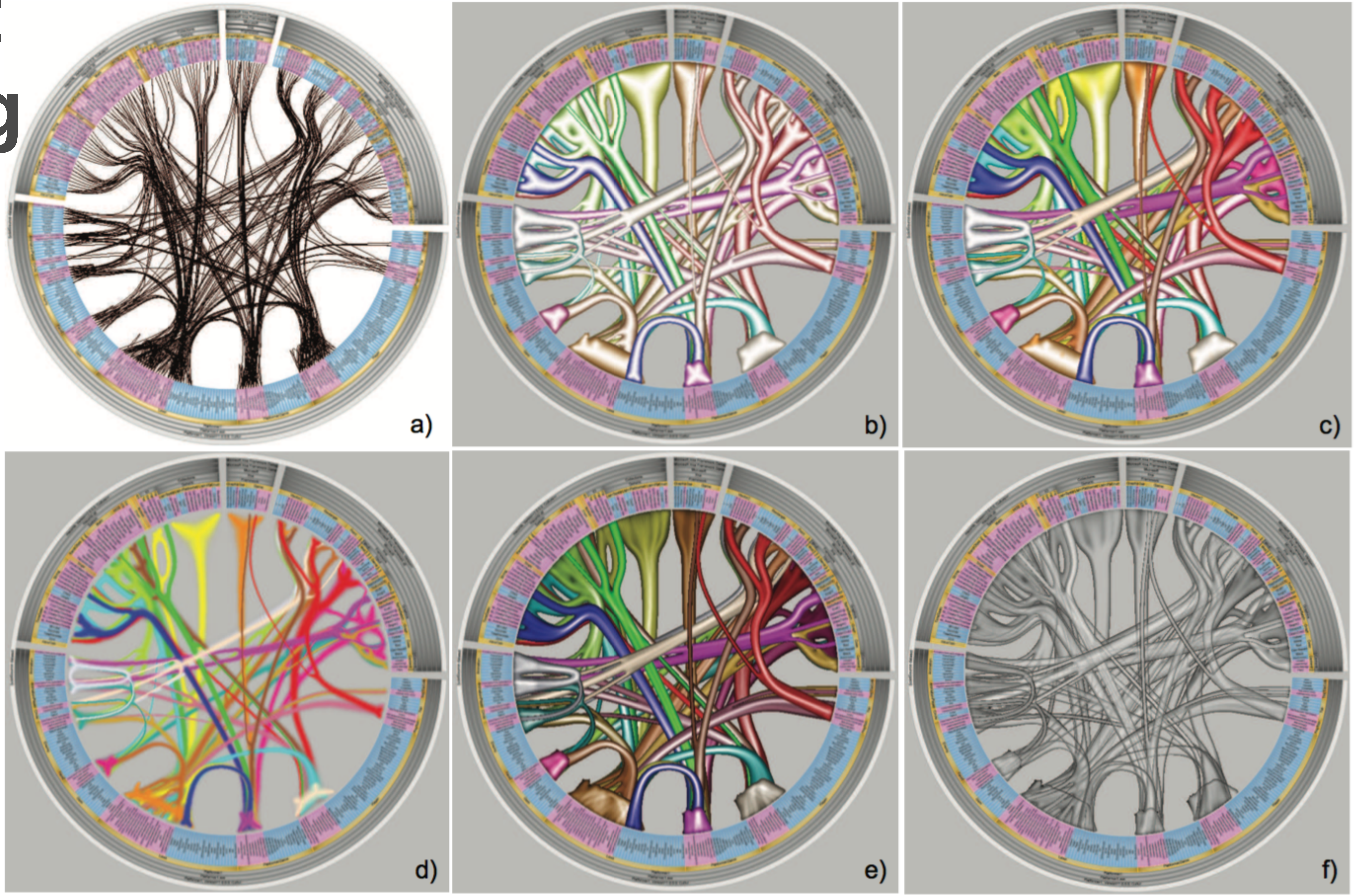

Figure 5: *Rendering styles: convex shapes (b), density-luminance (c), density-saturation (d), bi-level (e), and outlines (f).*

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Interaction: Digging Lens

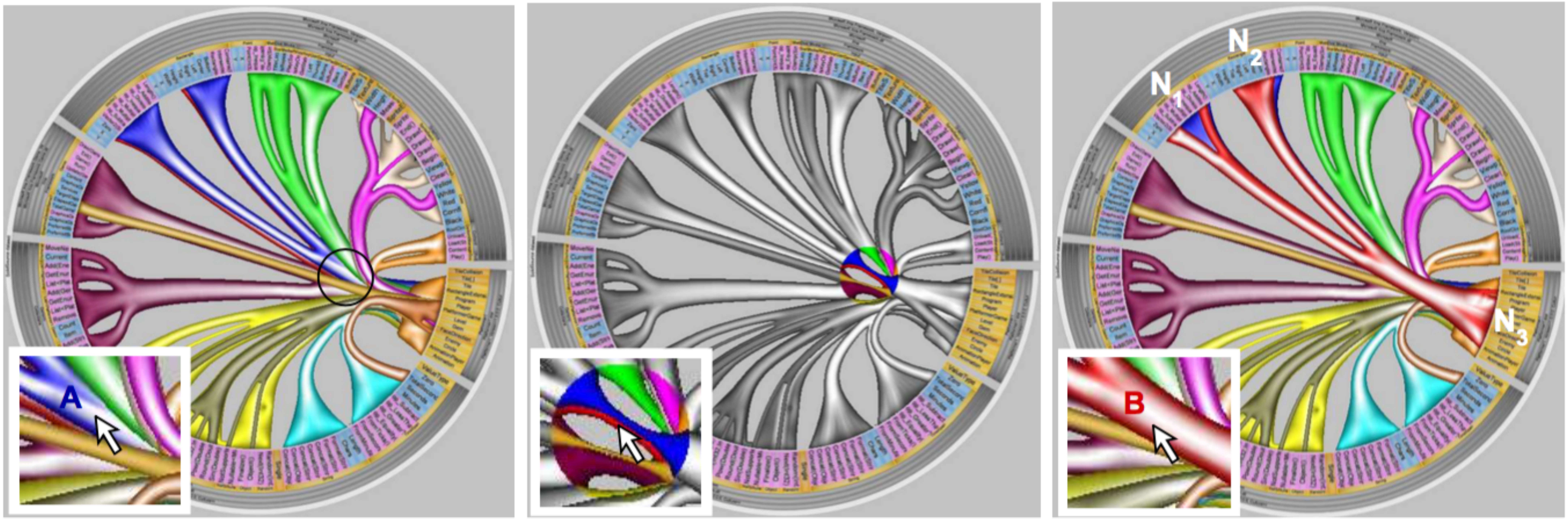

Figure 7: *The digging lens is used to interactively explore areas where shapes overlap. Insets show zoomed-in details.*

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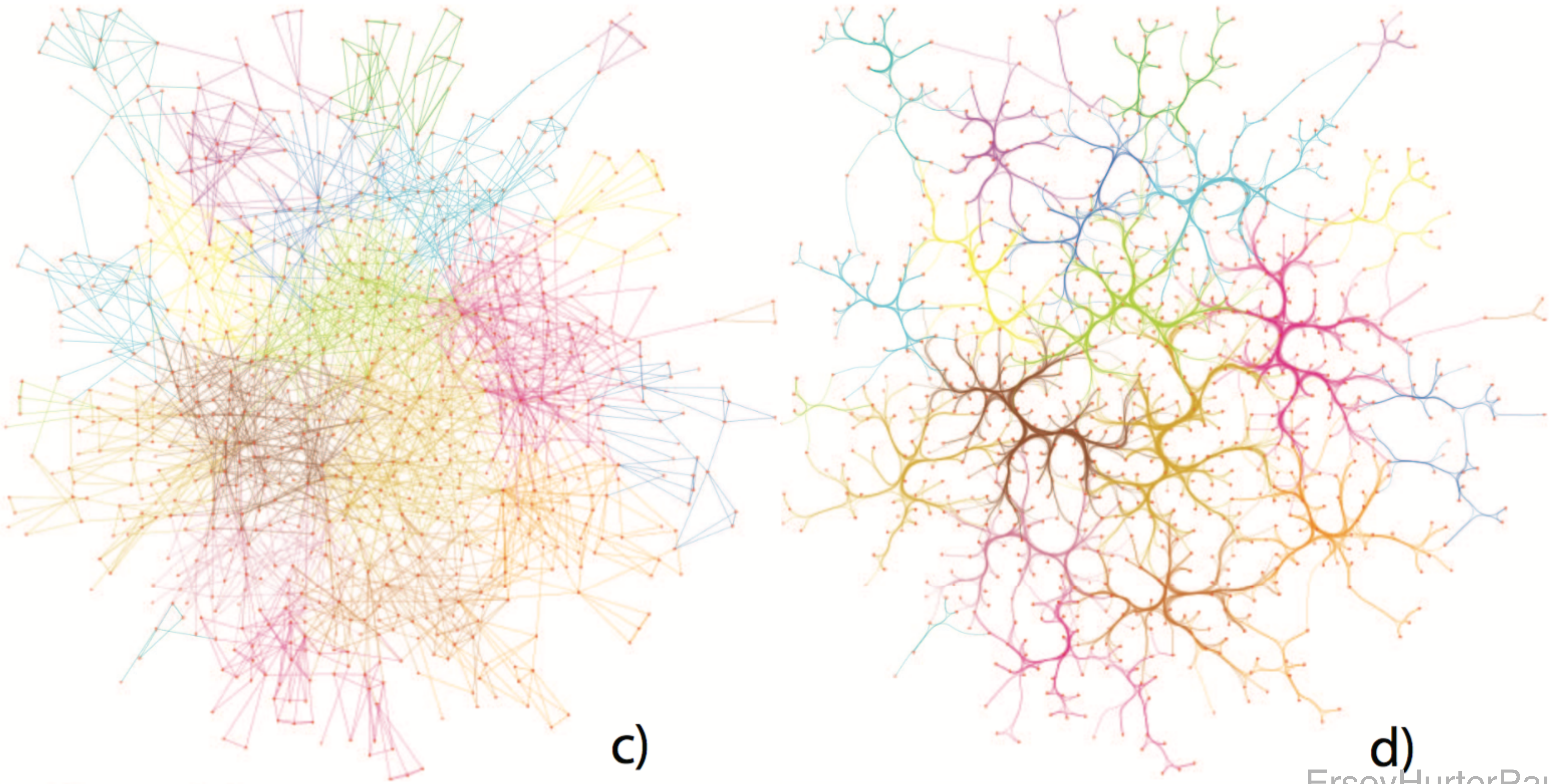
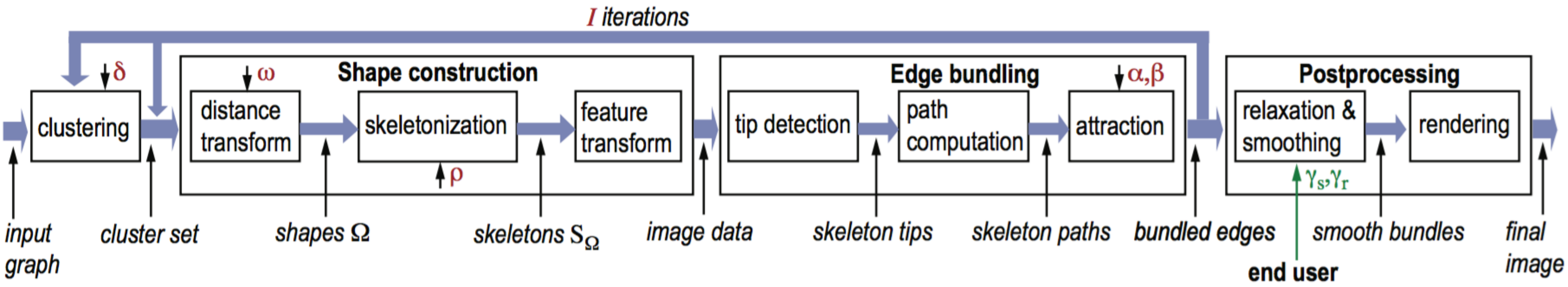
Skeleton-Based Edge Bundling

Skeleton-Based Edge Bundling



ErsoyHurterPaulovich2011



1. we *cluster* edges into groups, or clusters, C_i which have strong geometrical and optionally attribute-based similarity;

2. for each cluster C, we compute a thin shape Ω surrounding its edges using a distance-based method;

3. for each shape Ω, we compute its skeleton S_Ω and feature transform of the skeleton FT_S;

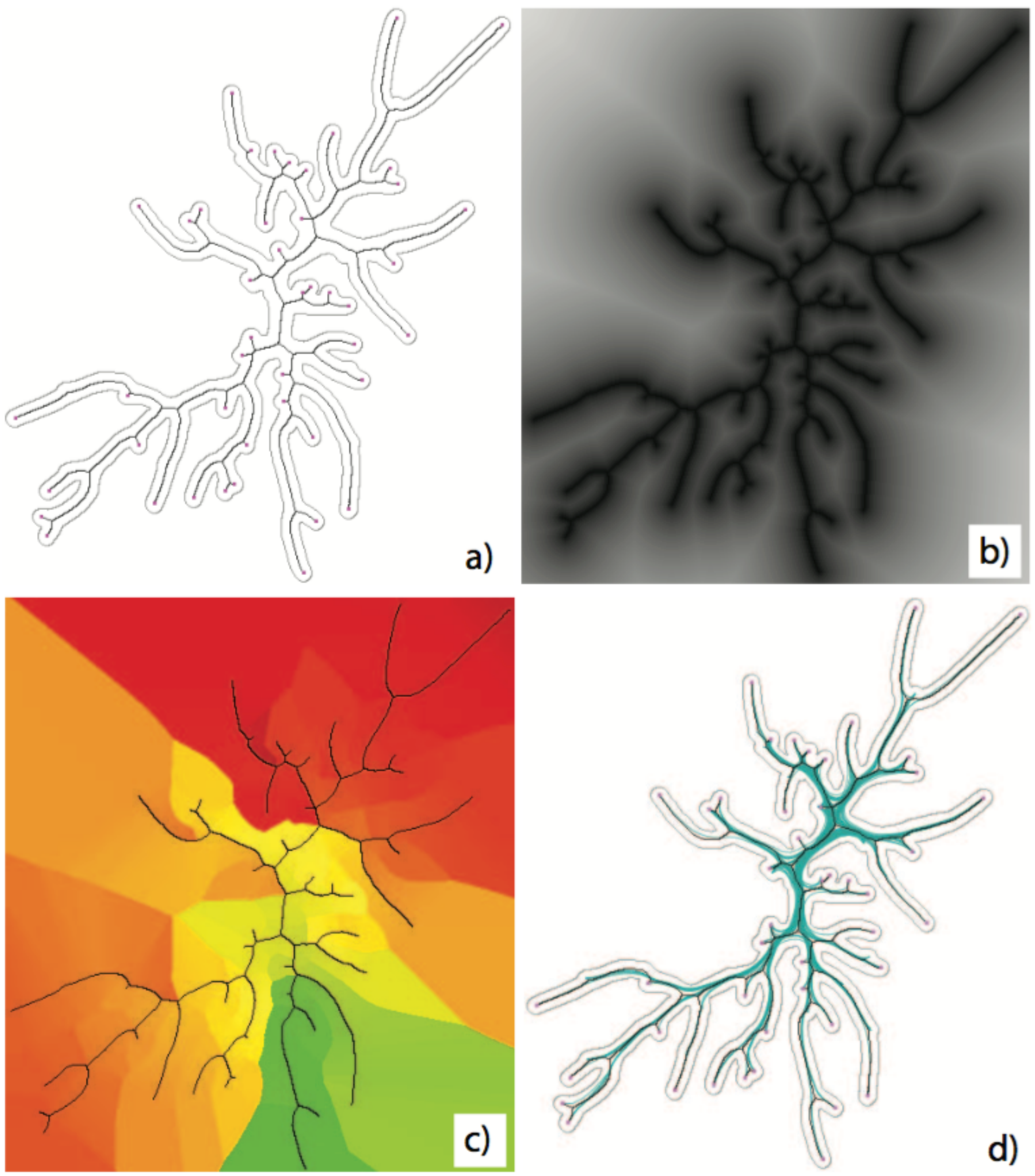
4. for each cluster C, we attract its edges towards S_Ω using FT_S;

5. we repeat the process from step 1 or step 2 until the desired bundling level is reached;

6. we perform a final smoothing and next render the graph using a cushion-like technique to help understanding bundle overlaps.

ErsoyHurterPaulovich2011

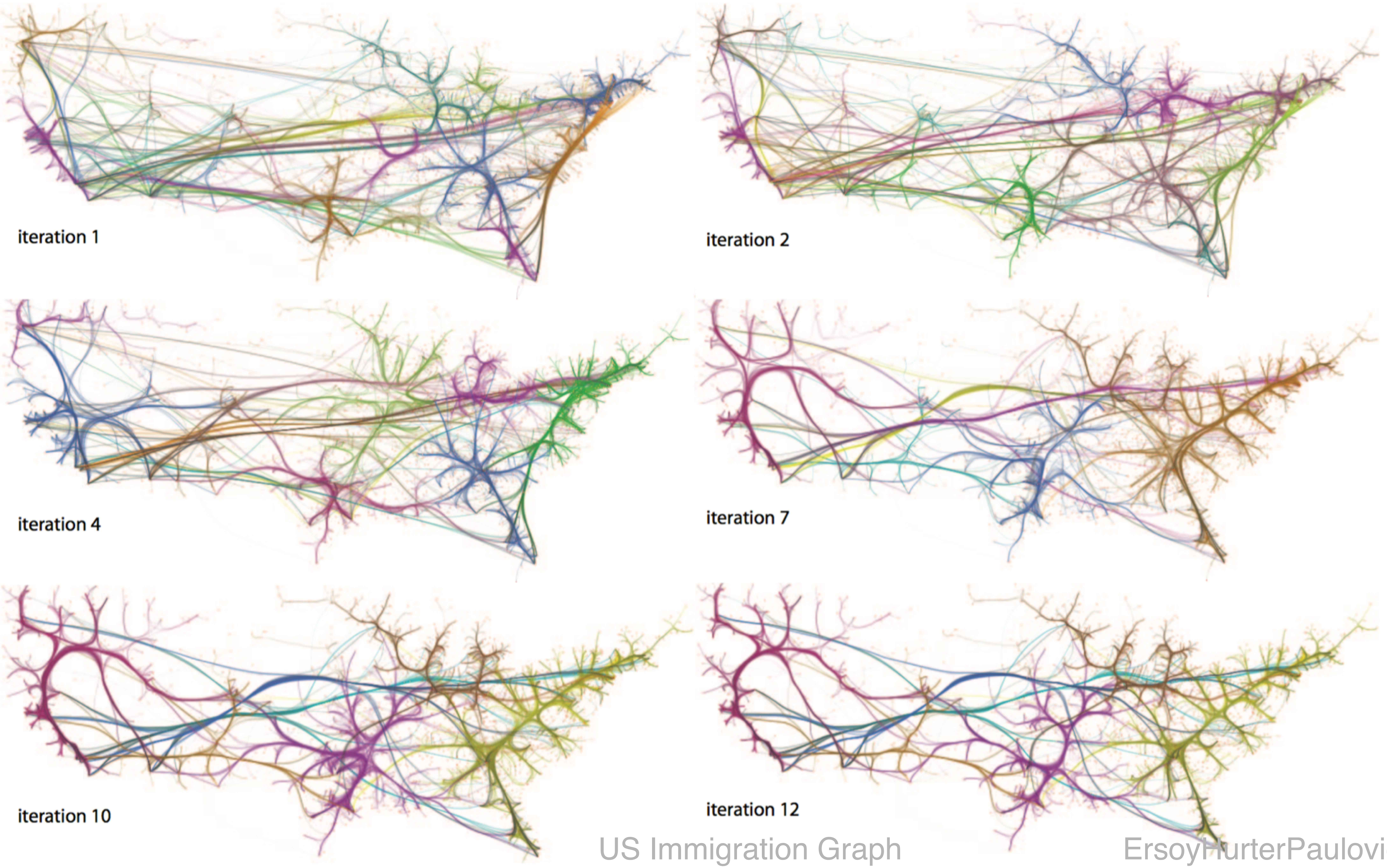
Shape Construction



ErsoyHurterPaulovich2011

Relaxation and Smoothing

ErsoyHurterPaulovich2011



US Immigration Graph

ErsoyHurterPaulovich2011

Comparison

Type	Technique	Computation	Advantages	Disadvantages
Flow Map Layout	[PXY*05]	low time complexity	1. intuitive 2. fast computation	1. not clear how to extend their method to general graphs [CZQ*08] 2. edge splits are binary [Hol06]
	[VBS11]	good computational costs	1. crossing-free [VBS11] 2. automated	limited usages to apply to general graphs
Hierarchical Edge Bundles	[Hol06]	good computational costs	1. significantly reduces visual clutter 2. suitable for software analysis	can only work on hierarchical structures
Geometry-Based Edge Bundling	[CZQ*08]	good computational costs	provides a clear visual pattern of densely bundled edges [LLCM12]	1. highly relies on the quality of the control meshes [LLCM12] 2. edges might create high curving variations [HvW09]
Force-Directed Edge Bundling	[HvW09]	high computational costs	able to be used on general graphs	1. difficult to add interactions because of high time complexity 2. does not effectively show the semantic properties of nodes and edges [KS10]
	[SHH11]	high computational costs	shows the direction, graph connectivity and weights in the bundled edges [SHH11]	difficult to add interactions because of high time complexity
Image-Based Edge Bundling	[TE10]	high computational costs	expresses coarse-scale structures [TE10]	limited by the resolution of the intermediate image [BSD13]
	[EHP*11]	massively accelerates bundling with GPU	1. Increased bundling speed 2. emphasis on structure of the bundled layout [EHP*11]	needs some works on bundle crossing minimization and node-edge overlap reduction [EHP*11]

[Zhou2017]

Graph-Theoretic Measures

Brain Connectivity Toolbox

https://sites.google.com/site/bctnet/measures/list

Incomplete list of measures

1 Degree and Similarity
2 Density and Rentian Scaling
3 Clustering and Community Structure
4 Assortativity and Core Structure
5 Paths and Distances
6 Efficiency and Diffusion
7 Centrality
8 Motifs

https://sites.google.com/site/bctnet/measures/list Brain Connectivity Toolbox

Modularity

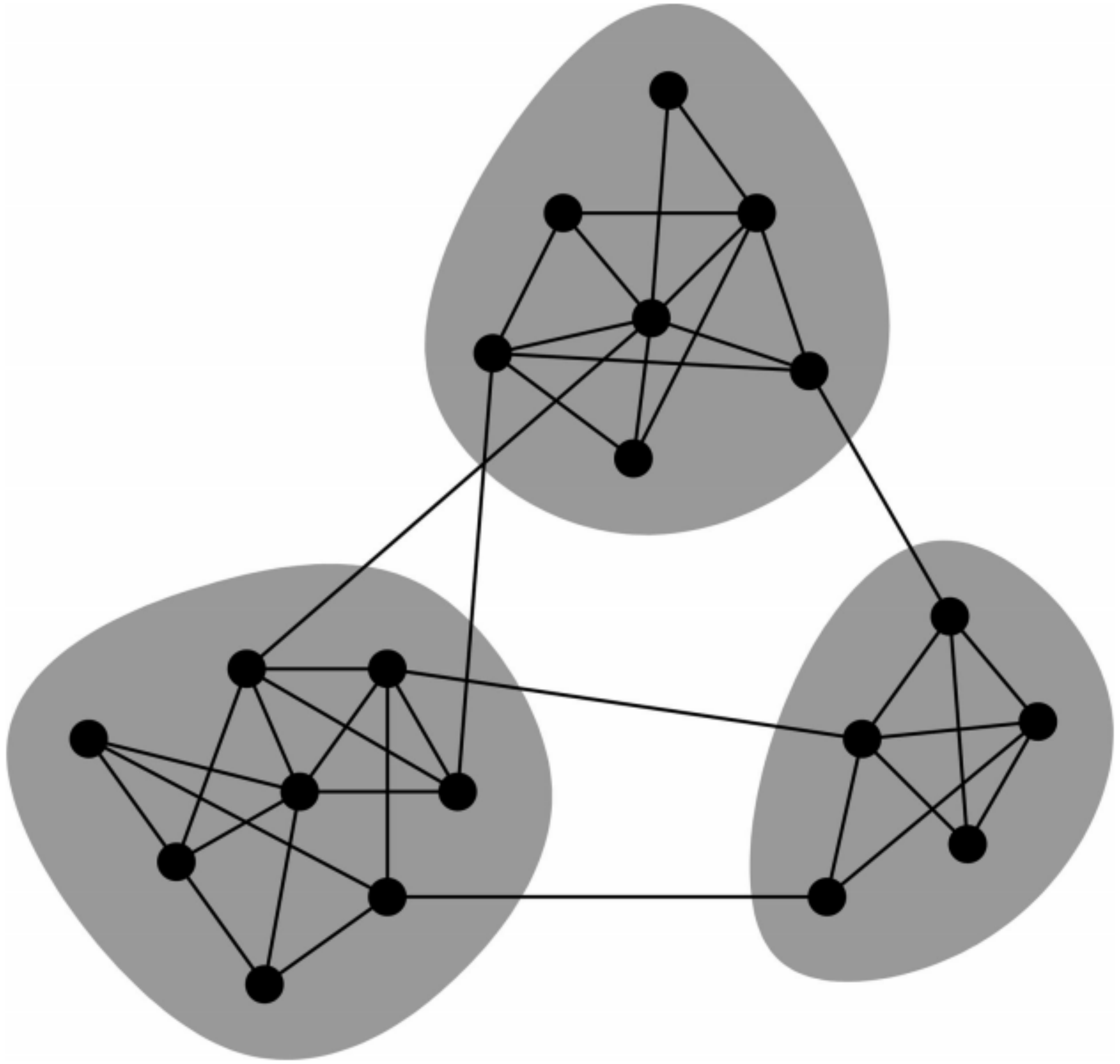


Fig. 1. The vertices in many networks fall naturally into groups or communities, sets of vertices (shaded) within which there are many edges, with only a smaller number of edges between vertices of different groups.

https://en.wikipedia.org/wiki/Modularity_(networks) Newman2006

Modularity

$$Q = \frac{1}{4m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right)(s_i s_j + 1) = \frac{1}{4m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right)s_i s_j, \quad [1]$$

$$Q = \frac{1}{4m}\mathbf{s}^T\mathbf{B}\mathbf{s}, \quad [2]$$

$$B_{ij} = A_{ij} - \frac{k_i k_j}{2m}, \quad [3]$$

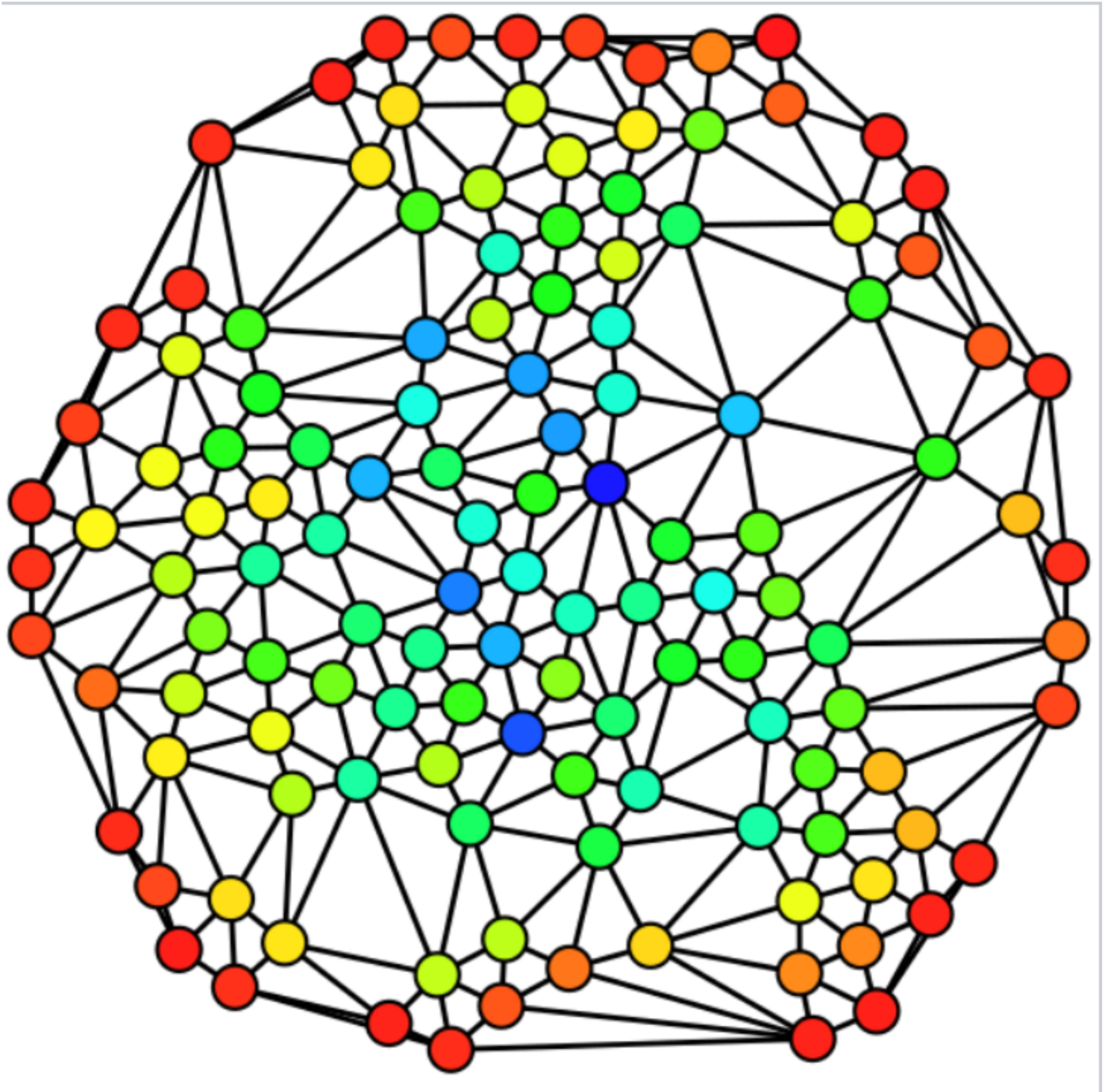
Newman2006

Modularity

- Network contains n vertices.
- For a division of the network into two groups:
  - si = 1 if vertex i belongs to group 1
  - si =-1 if it belongs to group 2
- Aij: adjacency matrix, number of edges between vertices i and j (0 or 1).
- ki, kj: degree of vertices i and j.
- ki*kj/2m: The expected number of edges between vertices i and j if
- edges are placed at random.
- m: total number of edges.
- Q: sum of Aij - ki*kj/2m over all pairs of vertices i,j that fall in the same group.

Newman2006

Betweenness Centrality



https://en.wikipedia.org/wiki/Betweenness_centrality

Betweenness Centrality

The betweenness centrality of a node v is given by the expression:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is the total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

https://en.wikipedia.org/wiki/Betweenness_centrality

Global Clustering Coefficient

$$C = \frac{3 \times \text{number of triangles}}{\text{number of connected triplets of vertices}}$$

https://en.wikipedia.org/wiki/Clustering_coefficient

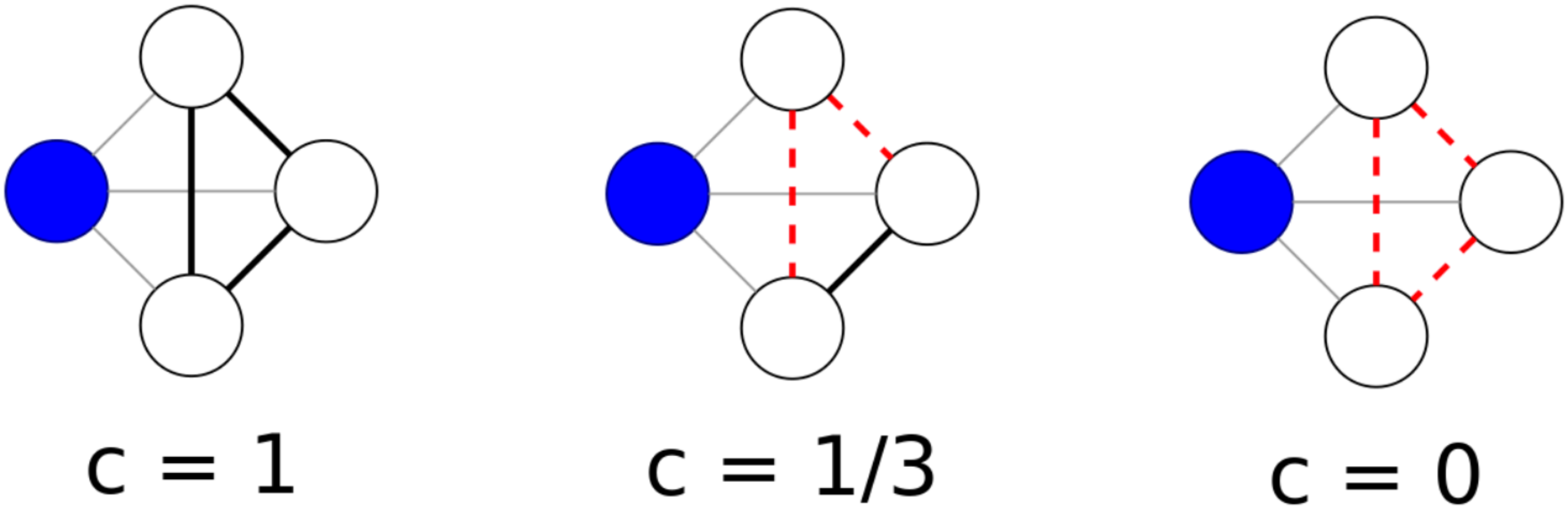
Local Clustering Coefficient

- The local clustering coefficient for a vertex is given by the proportion of links between the vertices within its neighborhood divided by the number of links that could possibly exist between them.

$$C_i = \frac{2|\{e_{jk} : v_j, v_k \in N_i, e_{jk} \in E\}|}{k_i(k_i - 1)}.$$

https://en.wikipedia.org/wiki/Clustering_coefficient

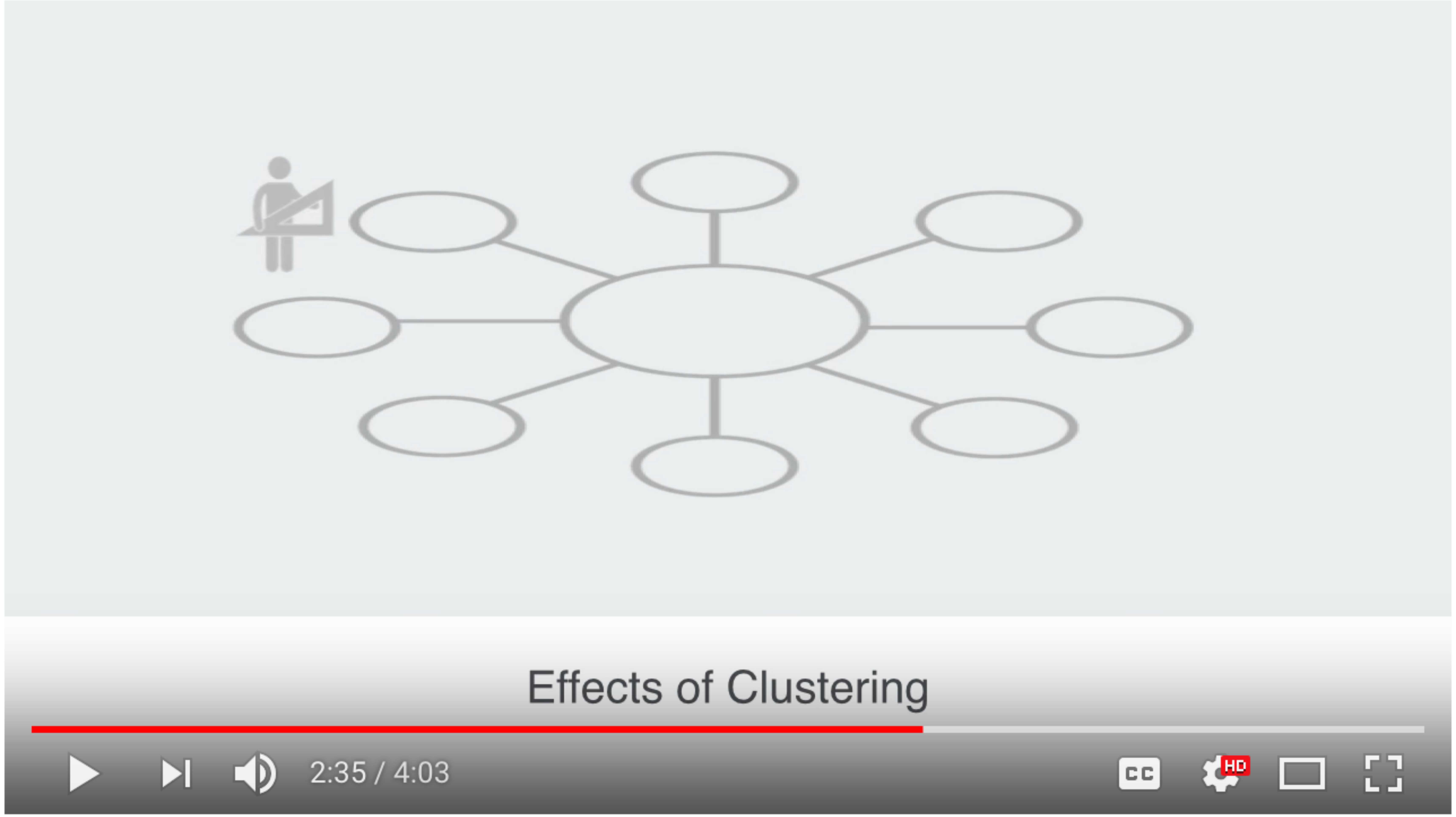
Local Clustering Coefficient



https://en.wikipedia.org/wiki/Clustering_coefficient

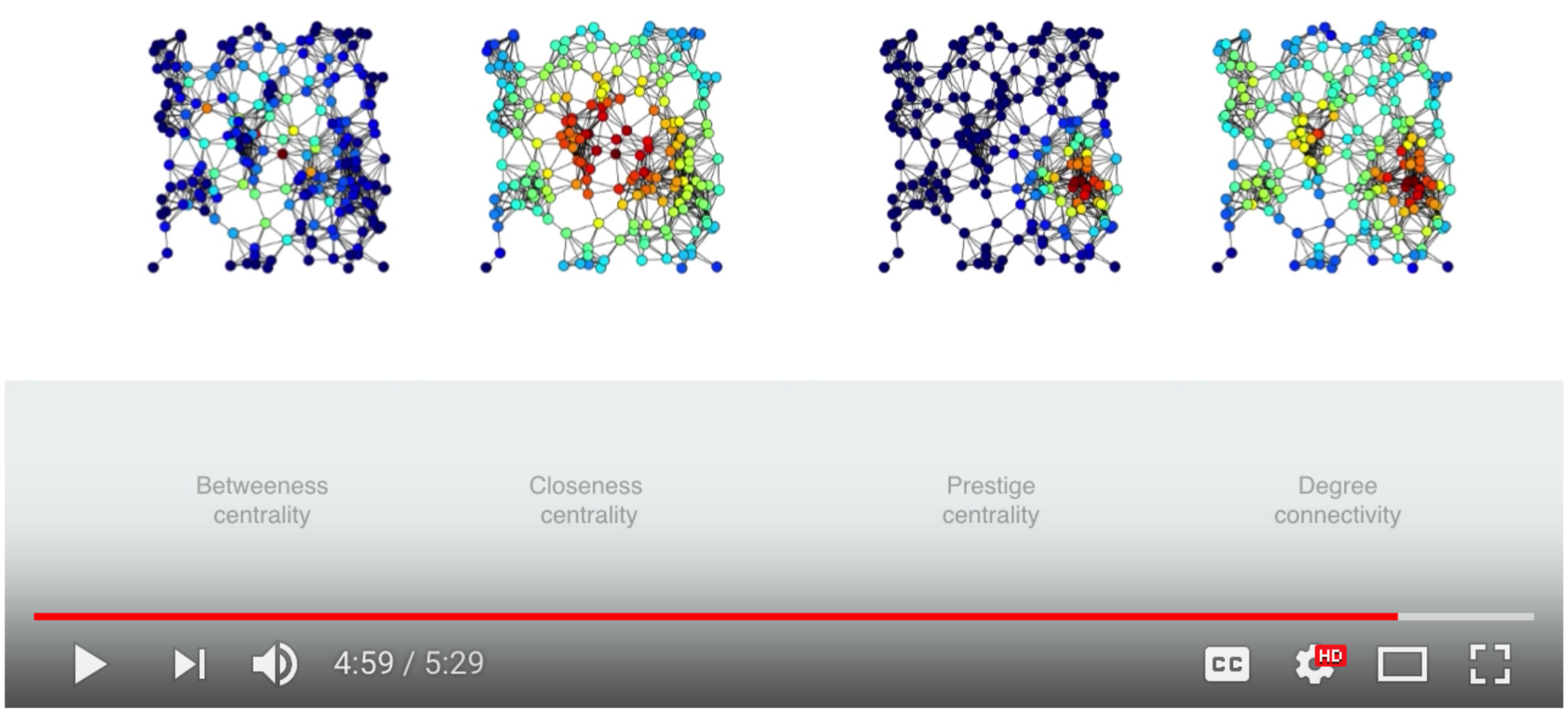
Additional Materials

Clustering Coefficient



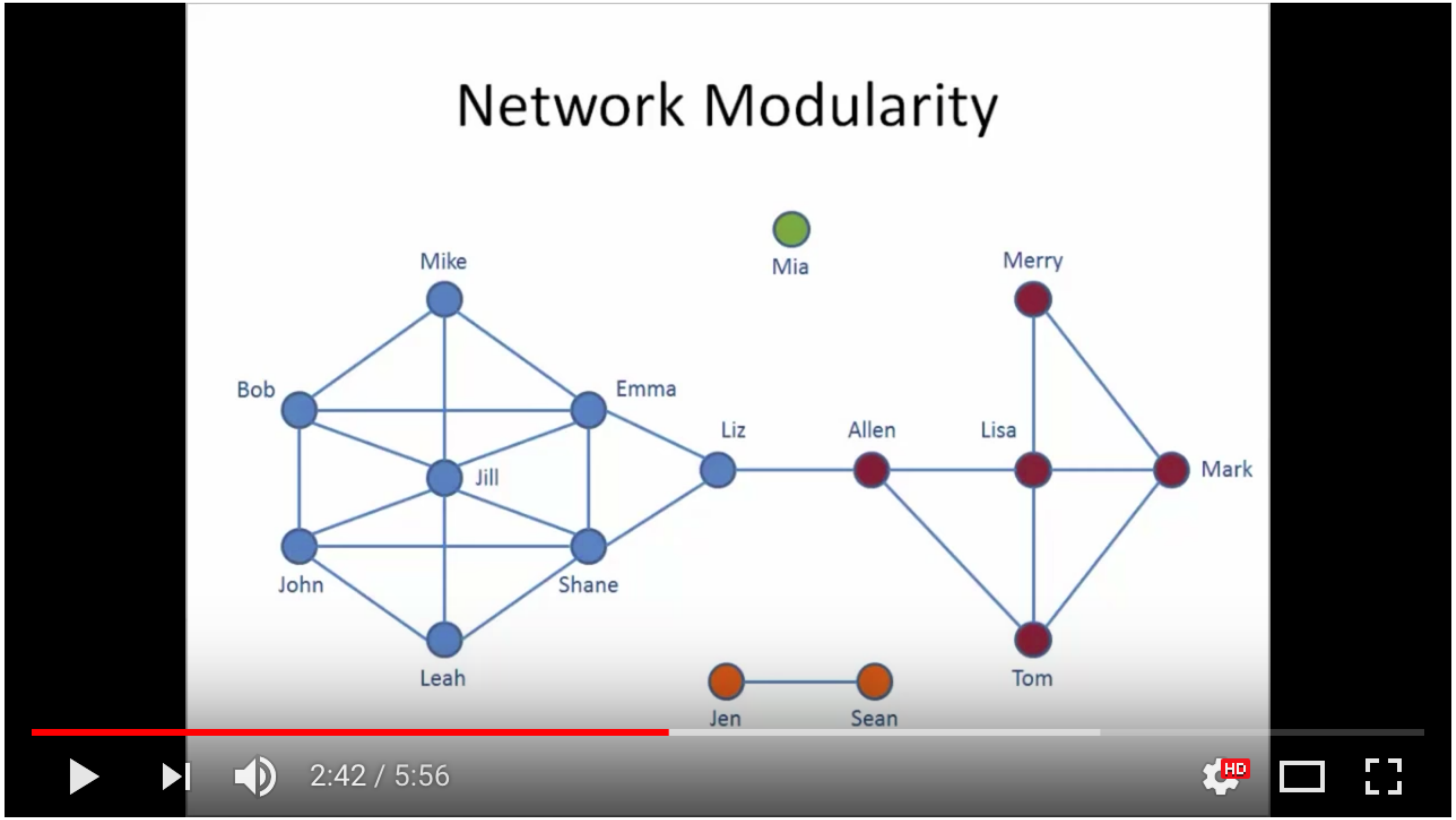
Network Clustering & Connectedness https://www.youtube.com/watch?v=2Oa7mef77nM

Centrality

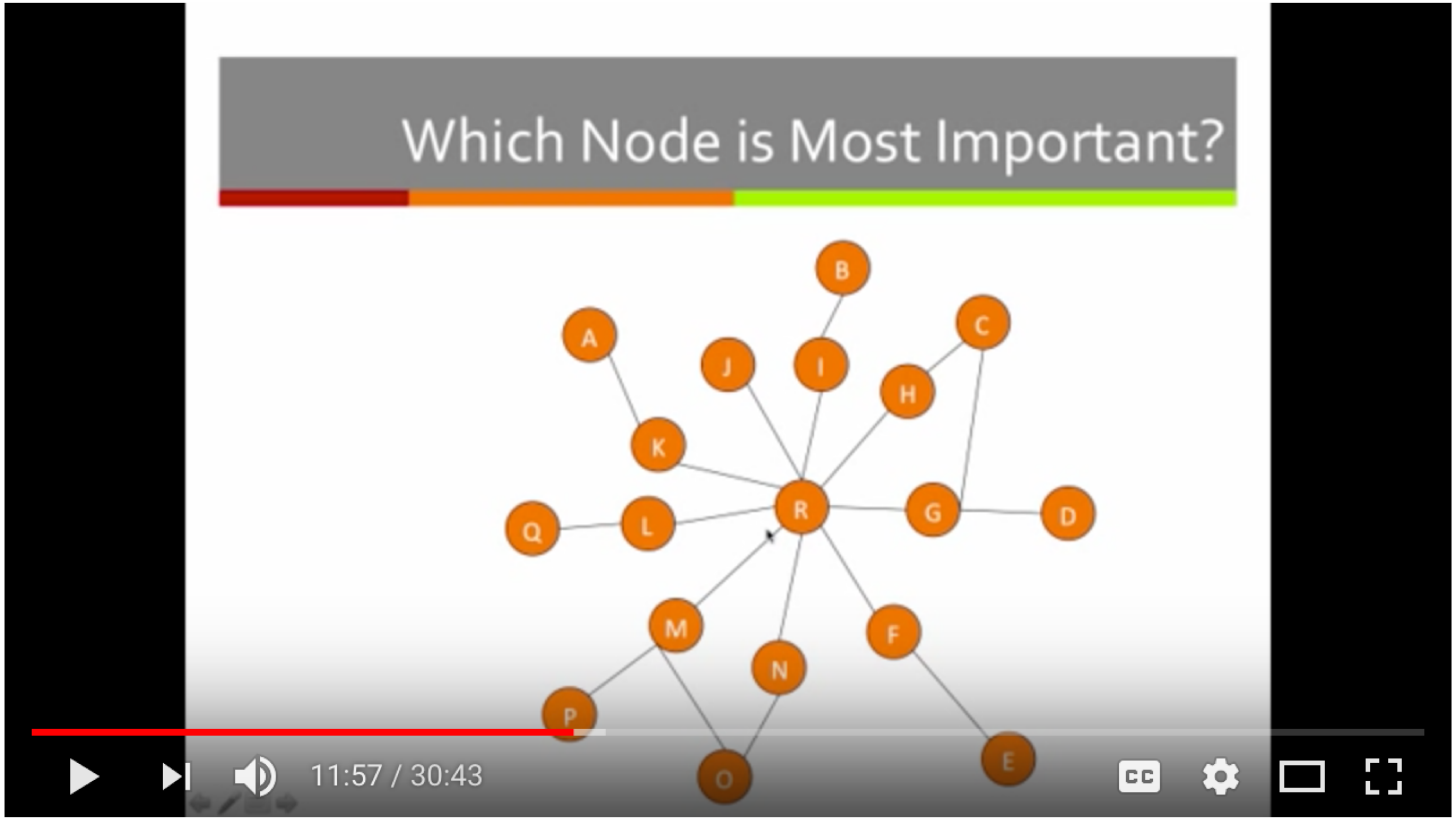


Network Centrality https://www.youtube.com/watch?v=NgUj8DEH5Tc

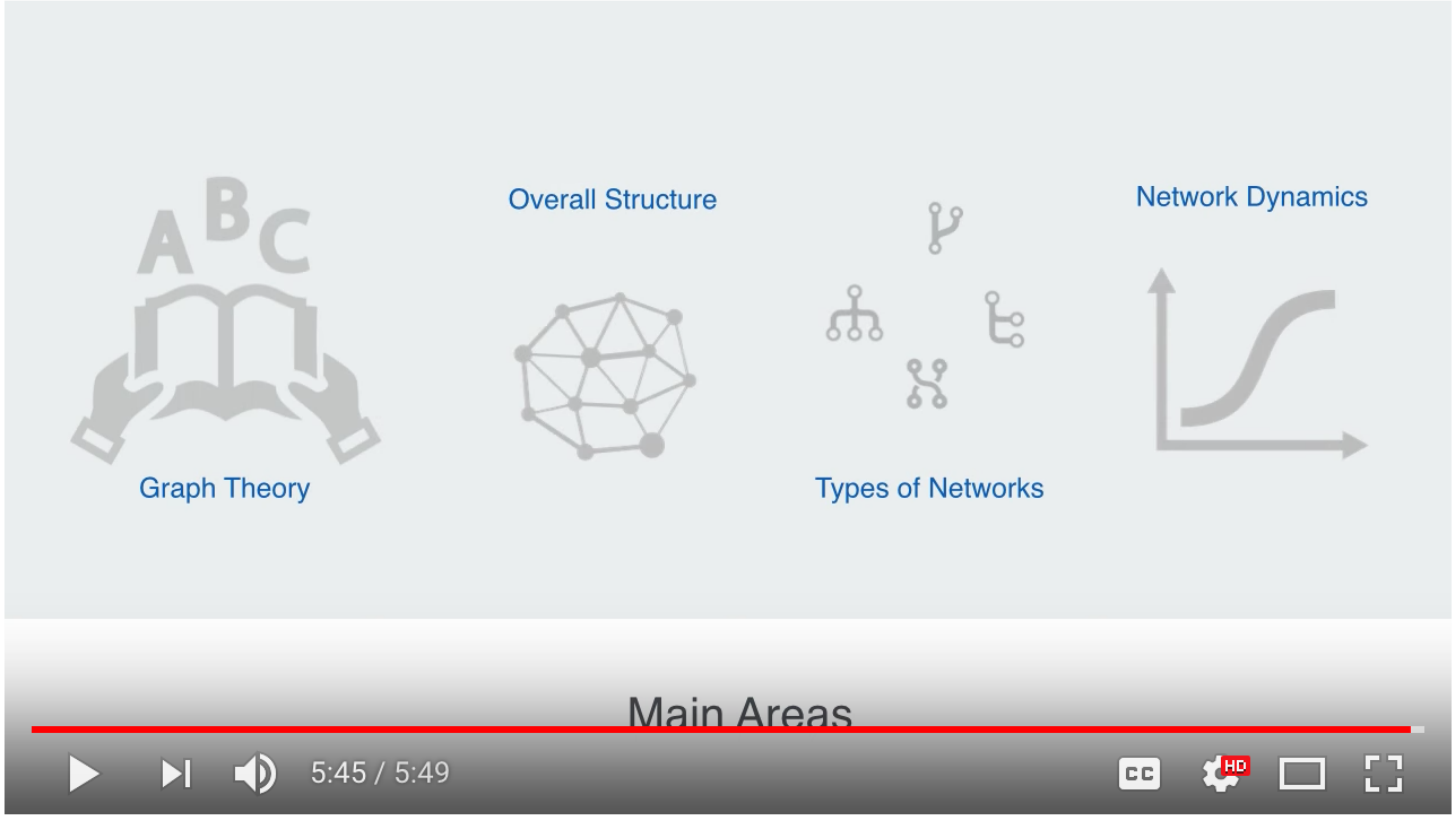
Network Modularity



https://www.youtube.com/watch?v=2_Q7uPAI34M



https://www.youtube.com/watch?v=89mxOdwPfxA



Network Theory Overview

https://www.youtube.com/watch?v=qFcuovfgPTc

https://www.youtube.com/watch?v=7HkXkAZye1Y&list=PLsJWgOB5mIMAuH3cHa-MXukX6-RPpDXgl

Coming Up:
Spectral Clustering
Label Propagation

Thanks!

Any questions?

You can find me at: **beiwang@sci.utah.edu**

CREDITS

Special thanks to all people who made and share these awesome resources for free:

- Presentation template designed by Slidesmash
- Photographs by unsplash.com and pexels.com
- Vector Icons by Matthew Skiles

Presentation Design

This presentation uses the following typographies and colors:

Free Fonts used:

http://www.1001fonts.com/oswald-font.html

https://www.fontsquirrel.com/fonts/open-sans

Colors used