

# Advanced Data Visualization

**CS 6965**

**Spring 2018**

**Prof. Bei Wang Phillips**

**University of Utah**



**Lecture 27**

# Graph-based ML

## ML on graphs

NV

# Announcement

- Final project presentation sign up!
- April 24 (Tuesday) 9:10 - 10:30 a.m, and April 27 (Friday) 8:00 to 10:00 a.m.

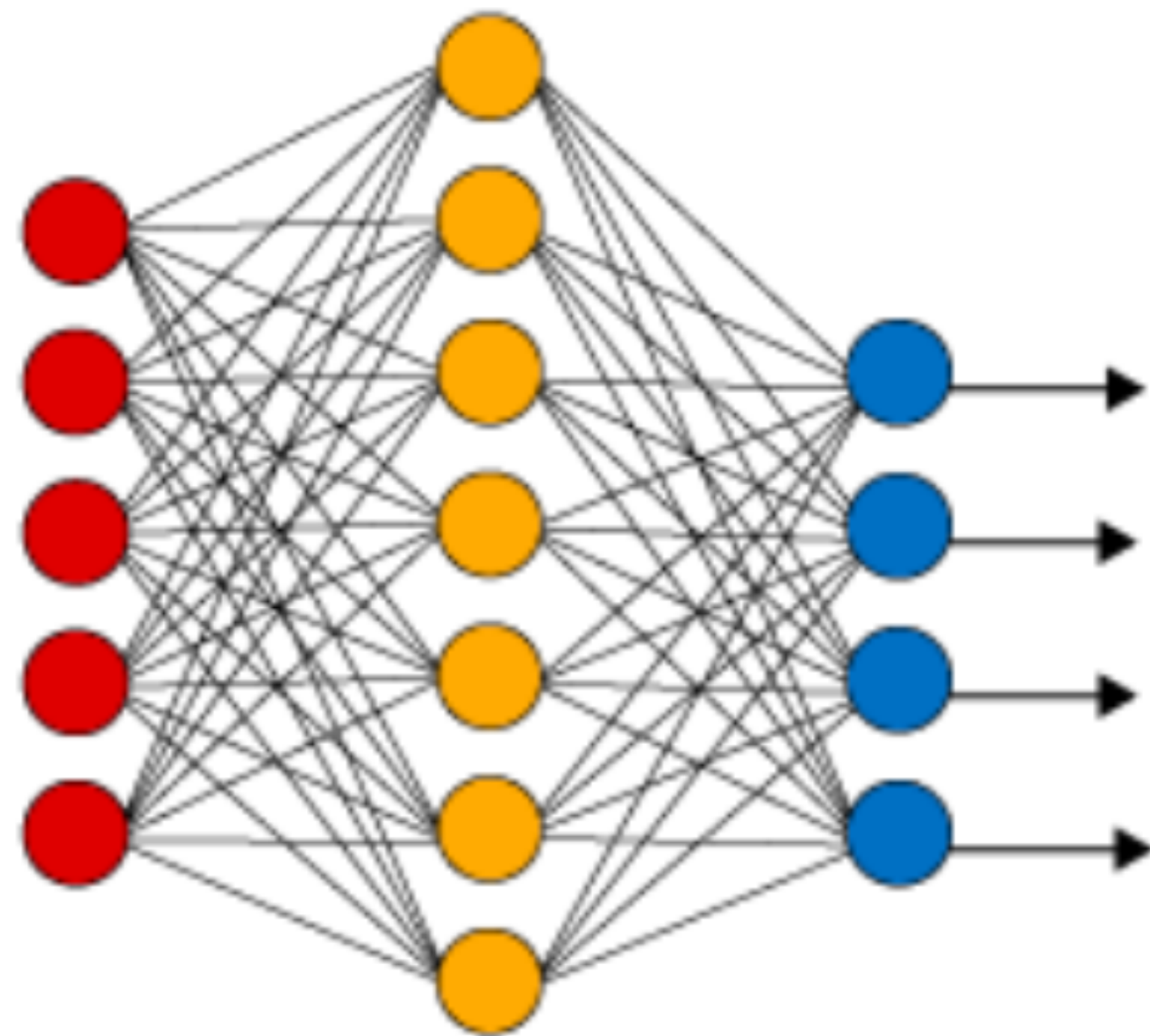
# Graph-Based ML

<https://research.googleblog.com/2016/10/graph-powered-machine-learning-at-google.html>

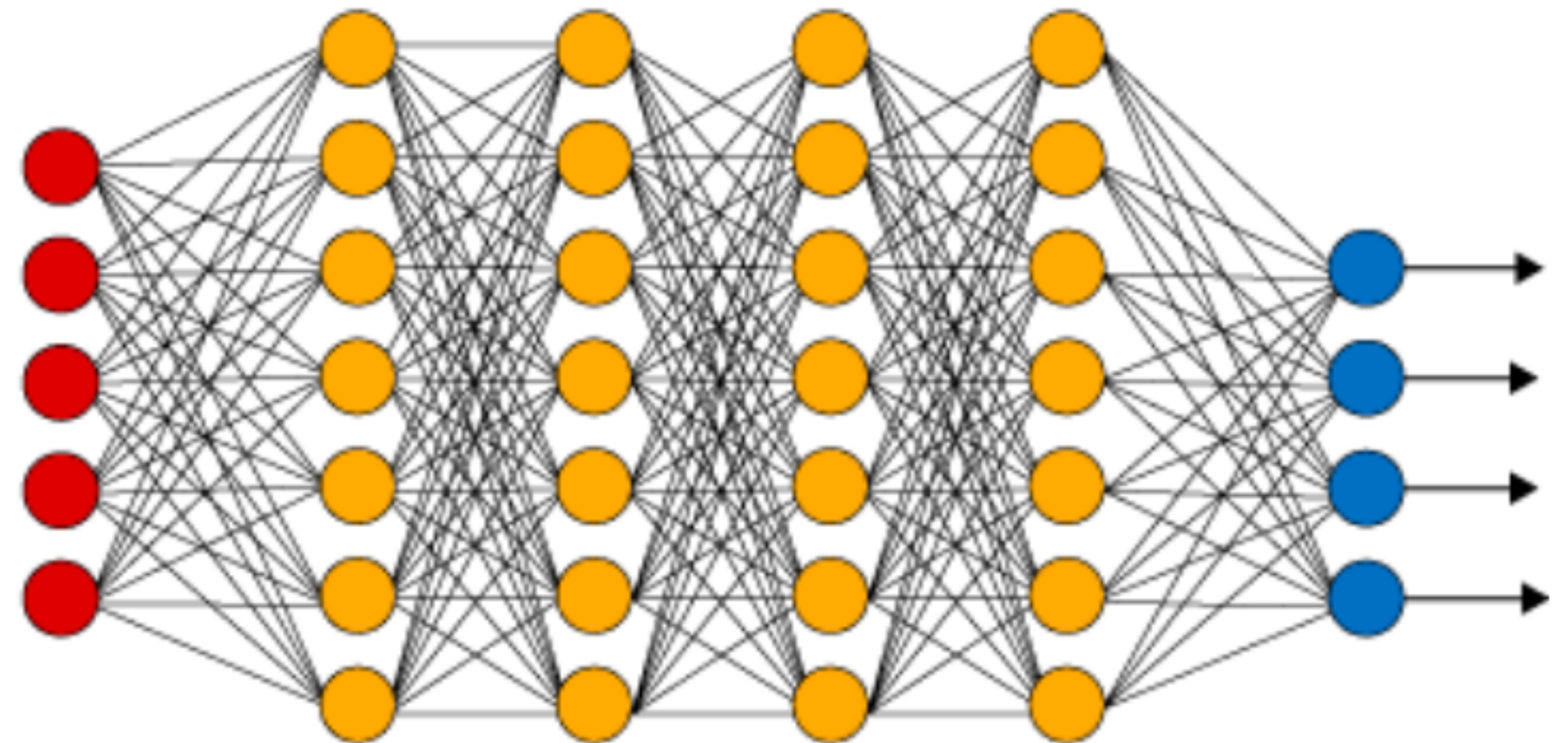
***What ML algorithms are based on graphs?***

# Deep Learning!

Simple Neural Network



Deep Learning Neural Network



● Input Layer    ● Hidden Layer    ● Output Layer

First train a system using labeled data with features and then apply the trained system to unlabeled data

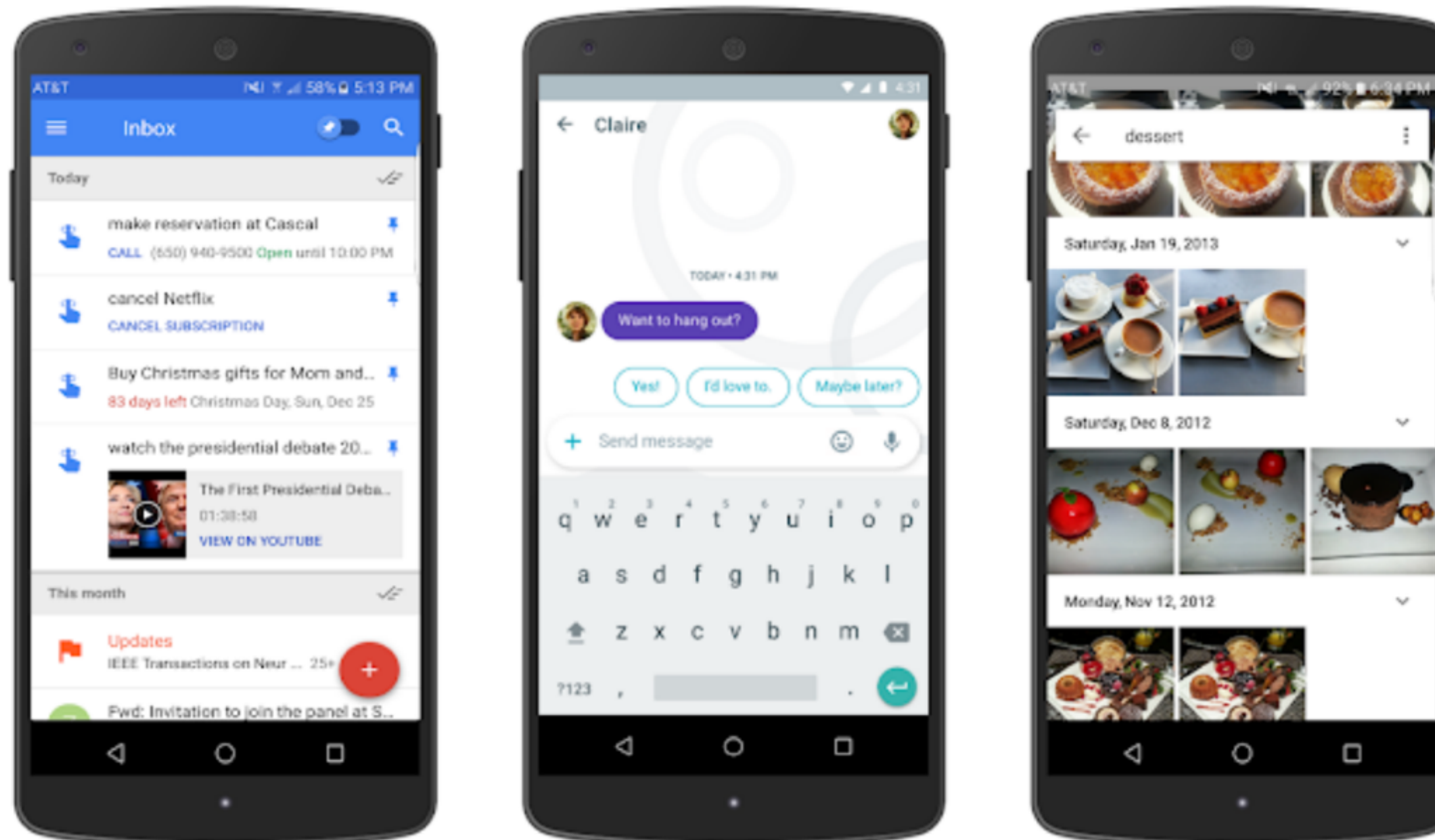
<http://www.global-engage.com/life-science/deep-learning-in-digital-pathology/>

# Graph-powered Machine Learning at Google

Thursday, October 06, 2016

Posted by Sujith Ravi, Staff Research Scientist, Google Research

Recently, there have been significant advances in [Machine Learning](#) that enable computer systems to solve complex real-world problems. One of those advances is Google's large scale, [graph-based](#) machine learning platform, built by the Expander team in Google Research. A technology that is behind many of the Google products and features you may use everyday, graph-based machine learning is a powerful tool that can be used to power useful features such as [reminders in Inbox](#) and [smart messaging in Allo](#), or used in conjunction with deep neural networks to power the latest image recognition system in [Google Photos](#).



[https://  
research.googleblog  
.com/2016/10/  
graph-powered-  
machine-learning-  
at-google.html](https://research.googleblog.com/2016/10/graph-powered-machine-learning-at-google.html)

# Graph-based semi-supervised learning

- Sparse training data
- Model labeled and unlabeled data jointly during learning, leveraging the underlying structure in the data
- Easily combine multiple types of signals (for example, relational information from Knowledge Graph along with raw features) into a single graph representation and learn over them.



***Graph-Based  
Semi-Supervised  
Learning***

# Semi-Supervised Learning Tutorial

Xiaojin Zhu

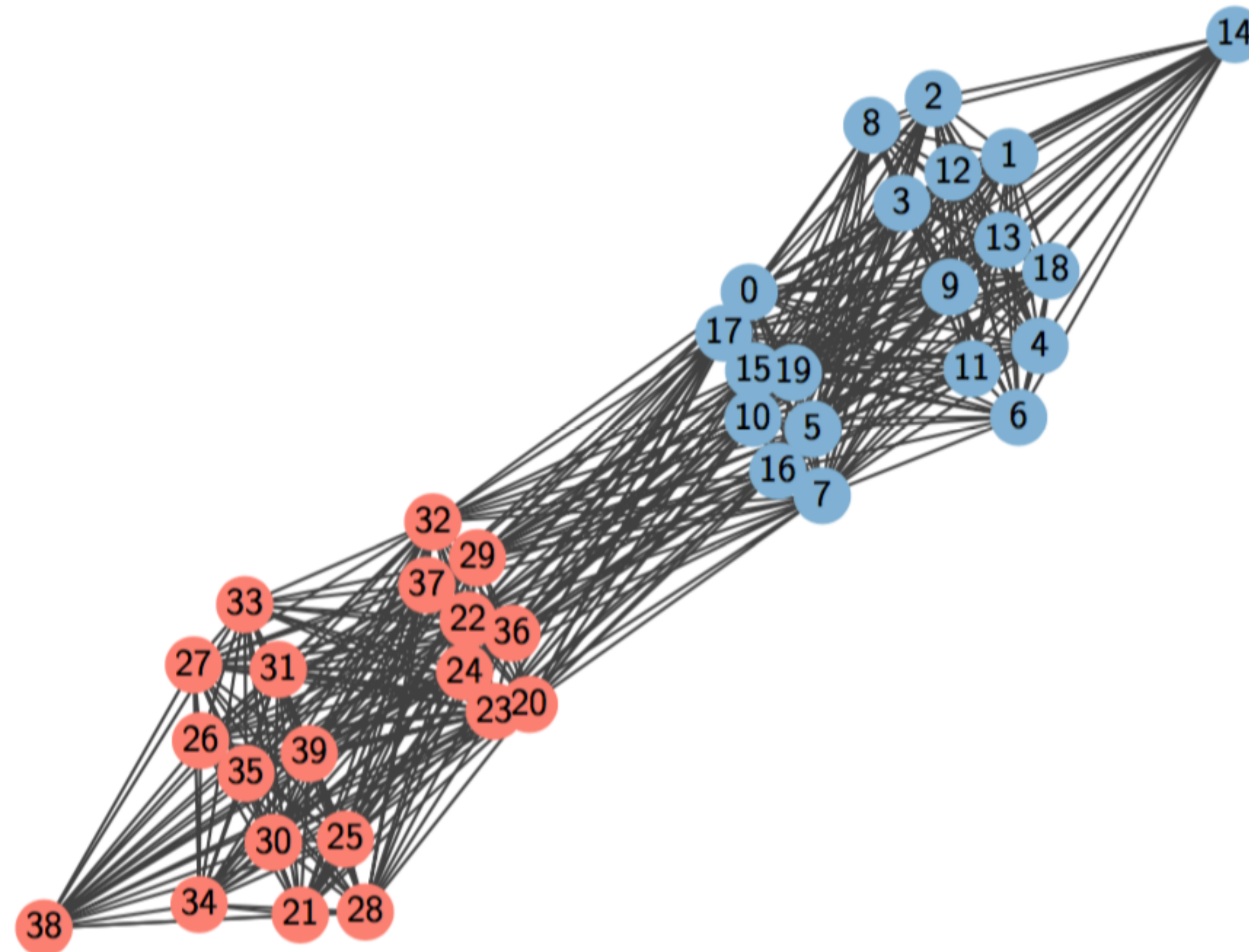
Department of Computer Sciences  
University of Wisconsin, Madison, USA

ICML 2007

<http://pages.cs.wisc.edu/~jerryzhu/icml07tutorial.html>

# *Spectral Clustering*

# Spectral Clustering



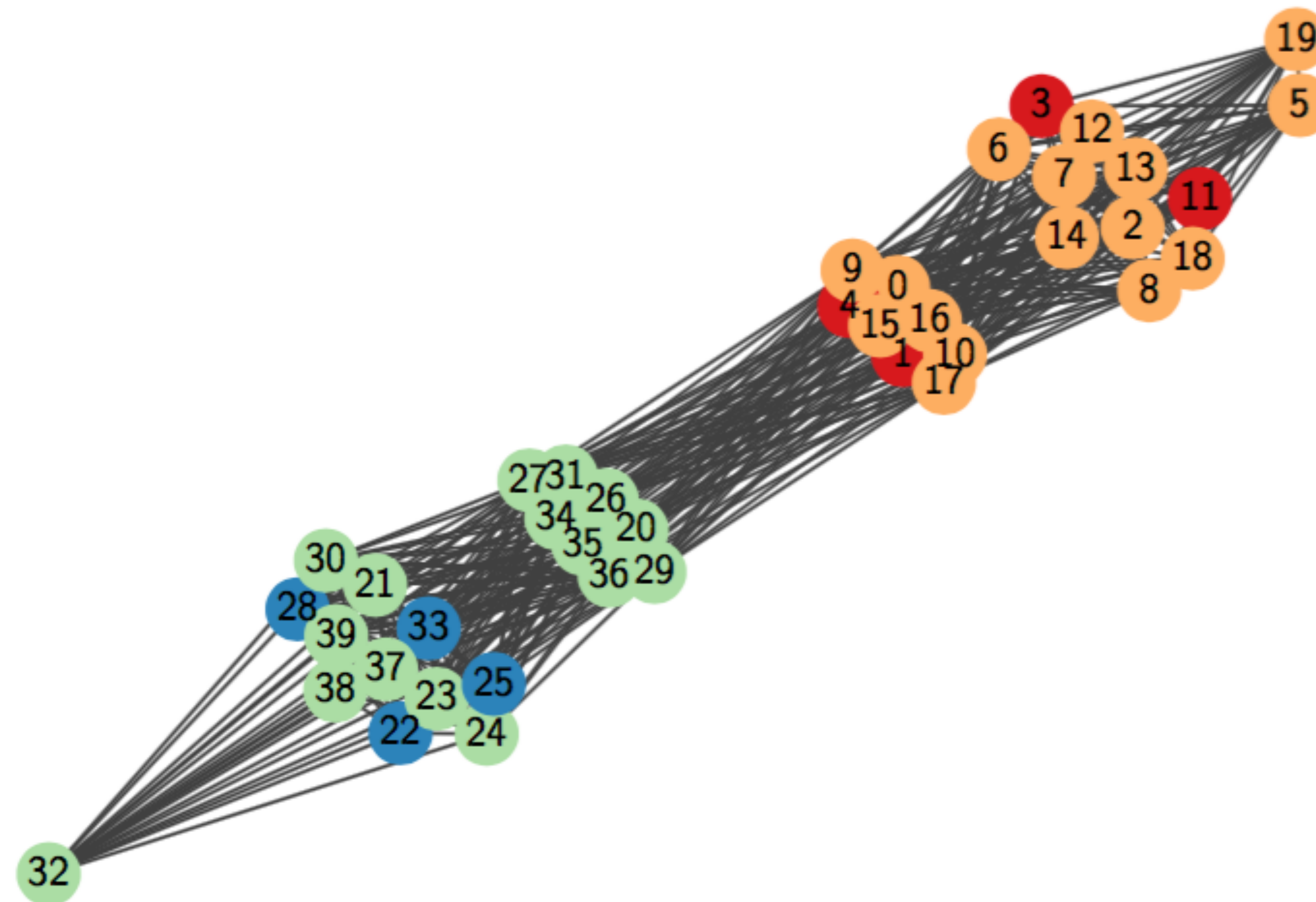
<https://arxiv.org/abs/1708.08436>

**Spectral clustering algorithm for graphs.** We use the Ng-Jordan-Weiss algorithm [39] to perform spectral clustering of graphs. Let  $n_0$  be the number of vertices in a graph. Recall the *affinity matrix*  $A \in \mathbb{R}^{n_0 \times n_0}$  is a matrix where  $A_{ij} (\geq 0)$  captures the affinity (i.e. measure of similarity) between vertex  $i$  and vertex  $j$ . In our setting,  $A_{ij}$  corresponds to the weight of edge  $e_{ij}$  in the diagonal edge weight matrix  $W_1$ . The spectral clustering algorithm in [39] can be summarized as follows:

1. Compute the diagonal matrix  $\Delta \in \mathbb{R}^{n_0 \times n_0}$  with diagonal elements  $\Delta_{ii}$  being the sum of  $A$ 's  $i$ -th row, that is,  $\Delta_{ii} = \sum_j A_{ij}$ .
2. Construct the matrix  $M = \Delta^{-1/2} A \Delta^{-1/2}$ .
3. Find  $u_1, u_2, \dots, u_k$ , the eigenvectors of  $M$  corresponding to the  $k$  largest eigenvalues (chosen to be orthogonal to each other in the case of repeated eigenvalues), and form the matrix  $X = [u_1 u_2 \dots u_k] \in \mathbb{R}^{n_0 \times k}$  by stacking the eigenvectors in columns.
4. Form the matrix  $Y$  from  $X$  by re-normalizing each of  $X$ 's rows to have unit length, that is,  $Y_{ij} = X_{ij} / \left( \sum_j X_{ij}^2 \right)^{1/2}$ .
5. Treating each row of  $Y$  as a point in  $\mathbb{R}^k$ , cluster them into  $k$  clusters via the  $k$ -means algorithm.
6. Finally, assign the original vertex  $v_i$  to cluster  $j$  if and only if row  $i$  of the matrix  $Y$  is assigned to cluster  $j$ .

# *Label Propagation*

# Label Propagation



<https://arxiv.org/abs/1708.08436>

**Label propagation on graphs.** We implement a simple version of the iterative label propagation algorithm [60] based on the notion of stochastic matrix (i.e. random walk matrix)  $P = \Delta^{-1}A$ , where  $A$  is the affinity matrix and  $\Delta$  is the diagonal matrix with diagonal elements  $\Delta_{ii} = \sum_j a_{ij}$  (as defined in Section 5.1).

The matrix  $P$  represents the probability of label transition. Given  $P$  and an initial label vector  $\mathbf{y}$ , we iteratively multiply the label vector  $\mathbf{y}$  by  $P$ . If the graph is *label-connected* (i.e. we can always reach a labeled vertex from any unlabeled one), then  $P^t$  converges to a stationary distribution, that is,  $P^t \mathbf{x} = \mathbf{x}$  for a large enough  $t$ .

Suppose there are two label classes  $\{+1, -1\}$ . Without loss of generality, assume that first  $l$  of the  $n$  vertices are assigned labels initially, represented as a length- $l$  vector  $\mathbf{y}_l$ . Given a graph  $G(V, E)$  and labels  $\mathbf{y}_l$ , the algorithm is given as:

1. Compute  $A$ ,  $\Delta$ , and  $P = \Delta^{-1}A$ .
2. Initialize  $\mathbf{y}^{(0)} = (\mathbf{y}_l, \mathbf{0})$ ,  $t = 0$ .
3. Repeat until convergence:

$$\mathbf{y}^{(t+1)} = P\mathbf{y}^{(t)},$$

$$\mathbf{y}_l^{(t+1)} = \mathbf{y}_l^{(t)}.$$

4. Return  $\text{sgn}(\mathbf{y}^{(t)})$ .



Consider  $P$  to be divided into blocks as follows:

$$P = \begin{pmatrix} P_{ll} & P_{lu} \\ P_{ul} & P_{uu} \end{pmatrix}$$

where  $l$  and  $u$  index the labeled and unlabeled vertices with the number of vertices  $n_0 = l + u$ . Let  $\mathbf{y} = (\mathbf{y}_l, \mathbf{y}_u)$  be the labels at convergence, then  $\mathbf{y}_u$  is given by :

$$\mathbf{y}_u = (I - P_{uu})^{-1} P_{ul} \mathbf{y}_l$$

As long as our graph is connected, it is also label-connected and  $(I - P_{uu})$  is non-singular. So we can directly compute the labels at convergence without going through the iterative process described above.

# Additional Reading

## Graph-based Semi-supervised Learning

**Zoubin Ghahramani**

**Department of Engineering  
University of Cambridge, UK**

`zoubin@eng.cam.ac.uk`

`http://learning.eng.cam.ac.uk/zoubin/`

**MLSS 2012  
La Palma**

`http://mlg.eng.cam.ac.uk/zoubin/talks/lect3ssl.pdf`



# Thanks!

Any questions?

You can find me at: [beiwang@sci.utah.edu](mailto:beiwang@sci.utah.edu)

# CREDITS

Special thanks to all people who made and share these awesome resources for free:

- ☐ Presentation template designed by [Slidesmash](#)
- ☐ Photographs by [unsplash.com](#) and [pexels.com](#)
- ☐ Vector Icons by [Matthew Skiles](#)

# Presentation Design

This presentation uses the following typographies and colors:

## Free Fonts used:

<http://www.1001fonts.com/oswald-font.html>

<https://www.fontsquirrel.com/fonts/open-sans>

## Colors used

